Light-front versus equal-time quantization in ϕ^4 theory

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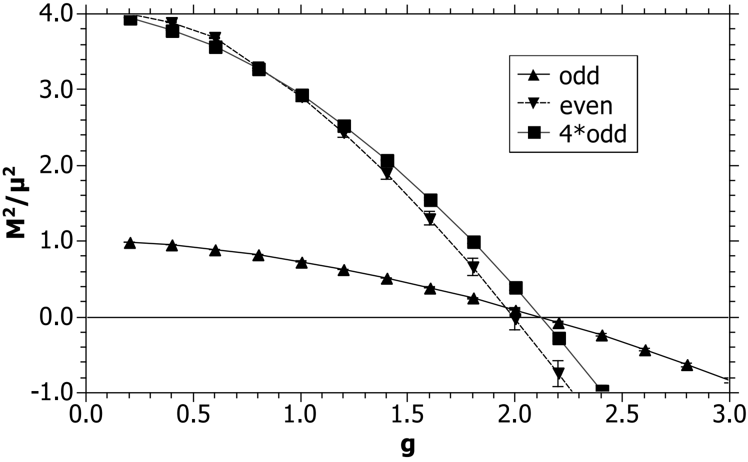
Overview

- systematic difference between light-front and equal-time values for critical coupling
  - pointed out at Frascati
  - resolved by considering difference in mass renormalizations [M. Burkardt, PRD **47**, 4628 (1993)]
  - arXiv:1606.00026 to appear in PRD
- probability for higher Fock sectors should grow dramatically as critical coupling is approached
  - not observed
  - sector-dependent mass did not help
  - seem to need basis with ∞ $\#$ of states
- details of methods provided by previous talk

Mass squared for unbroken phase



Critical coupling at $g = 2.1 \pm 0.05$.

Critical couplings compared: $\bar{g} \equiv \frac{\pi}{6} g$

Method	$\bar{g}_c$	Reported by
LF sym. polys.	1.1 ± 0.03	this work
DLCQ	1.38	Harindranath & Vary
Quasi-sparse eigenvector	2.5	Lee & Salwen
Density matrix	2.4954(4)	Sugihara
Lattice	$2.70 \left\{ \begin{array}{l} +0.025 \\ -0.013 \end{array} \right.$	Schaich & Loinaz
	2.79 ± 0.02	Bosetti *et al.*
Uniform matrix product	2.766(5)	Milsted *et al.*
Renorm. H trunc.	2.97(14)	Rychkov & Vitale

Mass renormalization

Bare mass renormalized by tadpole contributions in ET quantization but not in LF quantization
[M. Burkardt, PRD **47**, 4628 (1993)]

$$\mu_{\mathrm{LF}}^2 = \mu_{\mathrm{ET}}^2 + \lambda \left[\langle 0|\frac{\phi^2}{2}|0\rangle - \langle 0|\frac{\phi^2}{2}|0\rangle_{\mathrm{free}} \right].$$

The vev's of ϕ^2 resum the tadpole contributions; the subscript *free* indicates the vev with $\lambda = 0$.
$\rightarrow$ need to calculate vev's

$$\langle 0|\frac{\phi^2}{2}|0\rangle \rightarrow \frac{1}{2}\langle 0|\phi(\epsilon^+,\epsilon^-)\int_0^\infty dP \sum_n |\psi_n(P)\rangle\langle\psi_n(P)|\phi(0,0)|0\rangle.$$

$$\phi(\epsilon^+,\epsilon^-) = e^{i\mathcal{P}^-\epsilon^+/2}\phi(0,\epsilon^-)e^{-i\mathcal{P}^-\epsilon^+/2}.$$

Matrix elements

For the nth bound state

$$\langle\psi_n(P)|\phi(0,0)|0\rangle = \langle 0|\psi_{n1}^* a(P)\int \frac{dp}{\sqrt{4\pi p}} a^\dagger(p)|0\rangle = \frac{\psi_{n1}^*}{\sqrt{4\pi P}}$$

$$\begin{aligned}\langle 0|\phi(\epsilon^+,\epsilon^-)|\psi_n(P)\rangle \\ &= \langle 0|e^{i0\epsilon^+}\int \frac{dp}{\sqrt{4\pi p}} a(p) e^{-ip\epsilon^-/2} e^{-iM_n^2\epsilon^+/2P}\psi_{n1} a^\dagger(P)|0\rangle \\ &= \frac{\psi_{n1}}{\sqrt{4\pi P}} e^{-i(P\epsilon^- + M_n^2\epsilon^+/P)/2}\end{aligned}$$

For the one-particle free state

$$\langle 0|a(P)\phi(0,0)|0\rangle = \langle 0|a(P)\int \frac{dp}{\sqrt{4\pi p}} a^\dagger(p)|0\rangle = \frac{1}{\sqrt{4\pi P}}$$

$$\begin{aligned}\langle 0|\phi(\epsilon^+,\epsilon^-)a^\dagger(P)|0\rangle \\ &= \langle 0|e^{i0\epsilon^+}\int \frac{dp}{\sqrt{4\pi p}} a(p) e^{-ip\epsilon^-/2} e^{-i\mu^2\epsilon^+/2P} a^\dagger(P)|0\rangle \\ &= \frac{1}{\sqrt{4\pi P}} e^{-i(P\epsilon^- + \mu^2\epsilon^+/P)/2}\end{aligned}$$

vev's

$$\langle 0|\frac{\phi^2}{2}|0\rangle = \frac{1}{2}\sum_n \int_0^\infty dP \frac{|\psi_{n1}|^2}{4\pi P} e^{-i(P\epsilon^- + M_n^2\epsilon^+/P)/2}$$

and

$$\langle 0|\frac{\phi^2}{2}|0\rangle_{\rm free} = \frac{1}{2}\int_0^\infty dP \frac{1}{4\pi P} e^{-i(P\epsilon^- + \mu^2\epsilon^+/P)/2}.$$

With use of $1 = \sum_n |\psi_{n1}|^2$,

$$\langle 0|\frac{\phi^2}{2}|0\rangle - \langle 0|\frac{\phi^2}{2}|0\rangle_{\rm free} = \sum_n \frac{|\psi_{n1}|^2}{8\pi}\int_0^\infty \frac{dP}{P} e^{-iP\epsilon^-/2}\left[e^{-i\frac{M_n^2\epsilon^+}{2P}} - e^{-i\frac{\mu^2\epsilon^+}{2P}}\right]$$

Mass shift

$$\langle 0|\tfrac{\phi^2}{2}|0\rangle - \langle 0|\tfrac{\phi^2}{2}|0\rangle_{\rm free} = \sum_n \tfrac{|\psi_{n1}|^2}{4\pi}\left[K_0(M_n\sqrt{-\epsilon^2+i\eta}) - K_0(\mu\sqrt{-\epsilon^2+i\eta})\right].$$

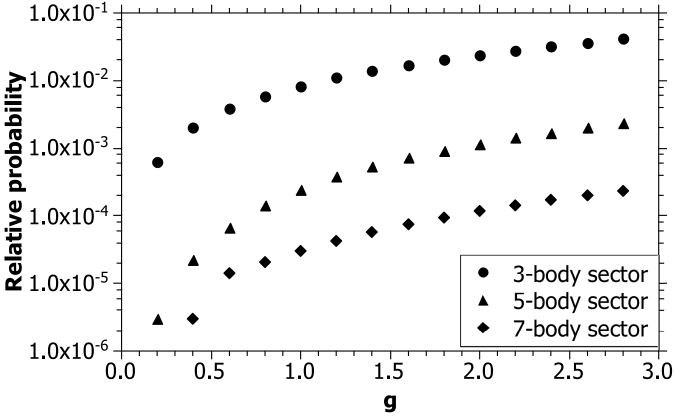
η is from a convergence factor; $K_0(z) \to -\ln(z/2) - \gamma$

$$\langle 0|\frac{\phi^2}{2}|0\rangle - \langle 0|\frac{\phi^2}{2}|0\rangle_{\rm free} = -\sum_n \frac{|\psi_{n1}|^2}{4\pi}\ln\frac{M_n}{\mu_{\rm LF}} \equiv -\Delta/4\pi,$$

$$\mu_{\rm LF}^2 = \mu_{\rm ET}^2 - \frac{\lambda}{4\pi}\Delta \quad \text{or} \quad \frac{\mu_{\rm ET}^2}{\mu_{\rm LF}^2} = 1 + g_{\rm LF}\Delta.$$

$$g_{\rm ET} = \frac{g_{\rm LF}}{\mu_{\rm ET}^2/\mu_{\rm LF}^2} = \frac{g_{\rm LF}}{(1+g_{\rm LF}\Delta)} \quad \text{and} \quad \frac{M^2}{\mu_{\rm ET}^2} = \frac{1}{1+g_{\rm LF}\Delta}\frac{M^2}{\mu_{\rm LF}^2}$$

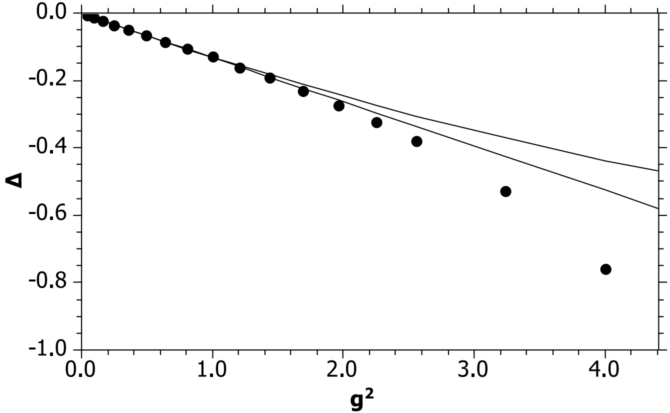
Relative probabilities



No indication of critical behavior at $g = 2.1$.

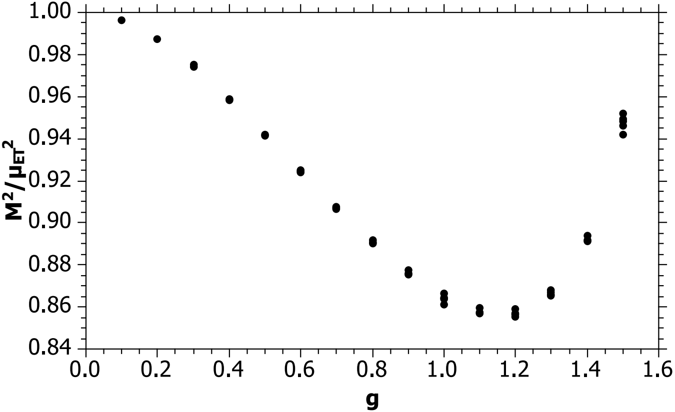
Computation of shift Δ will diverge $\sim |\psi_{11}|^2 \ln(M_1/\mu_{\rm LF})$.

Plot of shift



Points obtained as extrapolations in the basis size.
Lines are linear and quadratic fits to shifts below $g = 1$, extrapolated to the region of the critical coupling.

Plot of ET mass squared



Different points at the same g value correspond to different truncations of the basis.

Above $g = 1$, points begin to diverge.

Interpretation

- divergence from failure of $|\psi_{11}|$ to go to zero with M_1
  - the product $|\psi_{11}|^2 \ln(M_1/\mu_{\rm LF})$ then diverges
- can use extrapolations from below $g = 1$ to estimate Δ at the critical coupling
  - $\Delta(g = 2.1) = -0.47 \pm 0.12$
  - value is from the higher-order extrapolation, with the lower-order extrapolation used to indicate the error.
- from the latest equal-time value for the critical coupling [Rychkov & Vitale], $g_{\rm ETc} = \frac{6}{\pi}2.97 = 5.67$, we extract a shift of $(g_{\rm LFc}/g_{\rm ETc} - 1)/g_{\rm LFc} = -0.30$
- consistent with the estimated value of the shift

Summary

- can understand difference between ET and LF by taking different mass renormalizations into account
- calculation of the mass shift near critical coupling shows poor behavior of computed eigenstates
  - relative probabilities of higher Fock states do not show critical behavior
  - in fact, should $\to \infty$ as denominator $|\psi_{11}|^2 \to 0$
  - the original hypothesis, that sector-dependent masses would resolve the paradox, must be incorrect.
- correct representation near critical coupling apparently requires method without Fock-space truncation
  - coherent-state basis
  - LFCC with nontrivial valence state: $|1\rangle + |3\rangle$
    - ssc and jrh, PLB **711**, 417 (2012)