Symmetry breaking in light-front ϕ^4_2 theory

Work done in collaboration with S. Chabysheva.

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Introduction

- applying new method to ϕ^4 theory
  - expansion of wave functions in basis of multivariate symmetric polynomials
  - allows fine tuning of resolution, Fock sector by Fock sector
  - incorporates small-x behavior that captures integrable singularity
- exploring both symmetric and broken phases
  - symmetric phase emphasized in next talk by Chabysheva
  - here describe general method and consider broken phase

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Outline

- formulation of eigenvalue problem
  - Lagrangian and Hamiltonian
  - symmetric and asymmetric/broken
  - coupled systems of equations
  - symmetric polynomial method
  - sector-dependent mass
- results
  - symmetric and asymmetric/broken
  - comparison with Chang's duality
  - sector-dependent mass
- summary

Lagrangian and Hamiltonian

Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu_0^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

Hamiltonian:

$$\mathcal{H} = \pm\frac{1}{2}\mu^2 :\phi^2: + \frac{\lambda}{4!} :\phi^4:$$

Mode expansion:

$$\phi(x^+, x^-) = \int \frac{dp^+}{\sqrt{4\pi p^+}} \left\{ a(p^+)e^{-ipx} + a^\dagger(p^+)e^{ipx} \right\}$$

Commutation relation:

$$[a(p^+), a^\dagger(p'^+)] = \delta(p^+ - p'^+)$$

Asymmetric form

Shift by $U = \exp \int dp^+ [f(p^+) a^\dagger(p^+) - f^*(p^+) a(p^+)]$

$$U a(p^+) U^\dagger = a(p^+) + f^*(p^+).$$

Choose f to correspond to a zero mode
[Harindranath & Vary, PRD **37**, 3010 (1988)]

$f(p^+) = f^*(p^+) \equiv \sqrt{\pi p^+}\delta(p^+)\phi_s \Rightarrow U\phi U^\dagger = \phi + \phi_s$

and, with $\phi_s = \pm\sqrt{6\mu^2/\lambda}$,

$$U :\mathcal{H}^-: U^\dagger = -\frac{3}{2}\frac{\mu^4}{\lambda} + \frac{1}{2}(2\mu^2) :\phi^2: + \frac{\lambda\phi_s}{3!} :\phi^3: + \frac{\lambda}{4!} :\phi^4:$$

Substitution of the mode expansion yields

$$\mathcal{P}^- = \mathcal{P}^-_{11} + \mathcal{P}^-_{22} + \mathcal{P}^-_{13} + \mathcal{P}^-_{31} + \mathcal{P}^-_{12} + \mathcal{P}^-_{21},$$

Light-front Hamiltonian terms

$$\mathcal{P}^-_{11} = \int dp \frac{2\mu^2}{p} a^\dagger(p) a(p),$$

$$\mathcal{P}^-_{22} = \frac{\lambda}{4} \int \frac{dp_1 dp_2}{4\pi\sqrt{p_1 p_2}} \int \frac{dp'_1 dp'_2}{\sqrt{p'_1 p'_2}} \delta(p_1 + p_2 - p'_1 - p'_2) \times a^\dagger(p_1) a^\dagger(p_2) a(p'_1) a(p'_2)$$

$$\mathcal{P}^-_{13} = \frac{\lambda}{6} \int \frac{dp_1 dp_2 dp_3}{4\pi\sqrt{p_1 p_2 p_3 (p_1+p_2+p_3)}} a^\dagger(p_1 + p_2 + p_3) a(p_1) a(p_2) a(p_3),$$

$$\mathcal{P}^-_{12} = -\mu\sqrt{\frac{3\lambda}{2}} \int \frac{dp_1^+ dp_2^+}{\sqrt{4\pi p_1^+ p_2^+ (p_1^+ + p_2^+)}} a^\dagger(p_1^+ + p_2^+) a(p_1^+) a(p_2^+),$$

$$\mathcal{P}^-_{31} = (\mathcal{P}^-_{13})^\dagger, \quad \mathcal{P}^-_{21} = (\mathcal{P}^-_{12})^\dagger$$

Eigenstate

The eigenstate of $\mathcal{P}^-$, with eigenvalue M^2/P^+, can be expressed as an expansion

$$|\psi(P^+)\rangle = \sum_m P^{+\frac{m-1}{2}} \int \prod_i^m dy_i \delta(1-\sum_i^m y_i)\psi_m(y_i)|y_iP^+;P^+,m\rangle$$

in terms of Fock states

$$|y_iP^+;P^+,m\rangle = \frac{1}{\sqrt{m!}}\prod_{i=1}^m a^\dagger(y_iP^+)|0\rangle$$

with normalization

$$1 = \sum_m \int \prod_i^m dy_i \delta(1-\sum_i^m y_i)|\psi_m(y_i)|^2.$$

Coupled system of equations

$$\left\{\begin{array}{c} +\mu^2 \\ -\mu^2 \\ 2\mu^2 \end{array}\right\} \sum_i^m \frac{1}{y_i}\psi_m(y_i) + \frac{\lambda}{4\pi}\frac{m(m-1)}{4\sqrt{y_1 y_2}} \int \frac{dx_1 \psi_m(x_1, y_1+y_2-x_1, y_3, \ldots, y_m)}{\sqrt{x_1(y_1+y_2-x_1)}}$$

$$+ \frac{\lambda}{4\pi}\frac{m\sqrt{(m+2)(m+1)}}{6} \int \frac{dx_1 dx_2 \psi_{m+2}(x_1, x_2, y_1-x_1-x_2, y_2, \ldots, y_m)}{\sqrt{y_1 x_1 x_2 (y_1-x_1-x_2)}}$$

$$+ \frac{\lambda}{4\pi}\frac{(m-2)\sqrt{m(m-1)}}{6} \frac{\psi_{m-2}(y_1+y_2+y_3, y_4, \ldots, y_m)}{\sqrt{y_1 y_2 y_3 (y_1+y_2+y_3)}}$$

$$- \mu\sqrt{\frac{3\lambda}{8\pi}} m\sqrt{m+1} \int \frac{dx_1 \psi_{m+1}(x_1, y_1-x_1, y_2, \ldots, y_m)}{\sqrt{x_1(y_1-x_1)y_1}}$$

$$- \mu\sqrt{\frac{3\lambda}{8\pi}} (m-1)\sqrt{m} \frac{\psi_{m-1}(y_1+y_2, y_3, \ldots, y_m)}{\sqrt{y_1 y_2 (y_1+y_2)}} = M^2 \psi_m(y_i)$$

Symmetric polynomials

$$\psi_m(y_1,\ldots,y_m)=\sqrt{y_1y_2\cdots y_m}\sum_{ni}c_{ni}^{(m)}P_{ni}^{(m)}(y_1,\ldots,y_m)$$

$$\sum_{n'i'}\left[\left\{\begin{array}{l}+1\\-1\\2\end{array}\right\}T_{ni,n'i'}^{(m)}+gV_{ni,n'i'}^{(m,m)}\right]c_{n'i'}^{(m)}$$

$$+g\sum_{n'i'}V_{ni,n'i'}^{(m,m+2)}c_{n'i'}^{(m+2)}+g\sum_{n'i'}V_{ni,n'i'}^{(m,m-2)}c_{n'i'}^{(m-2)}$$

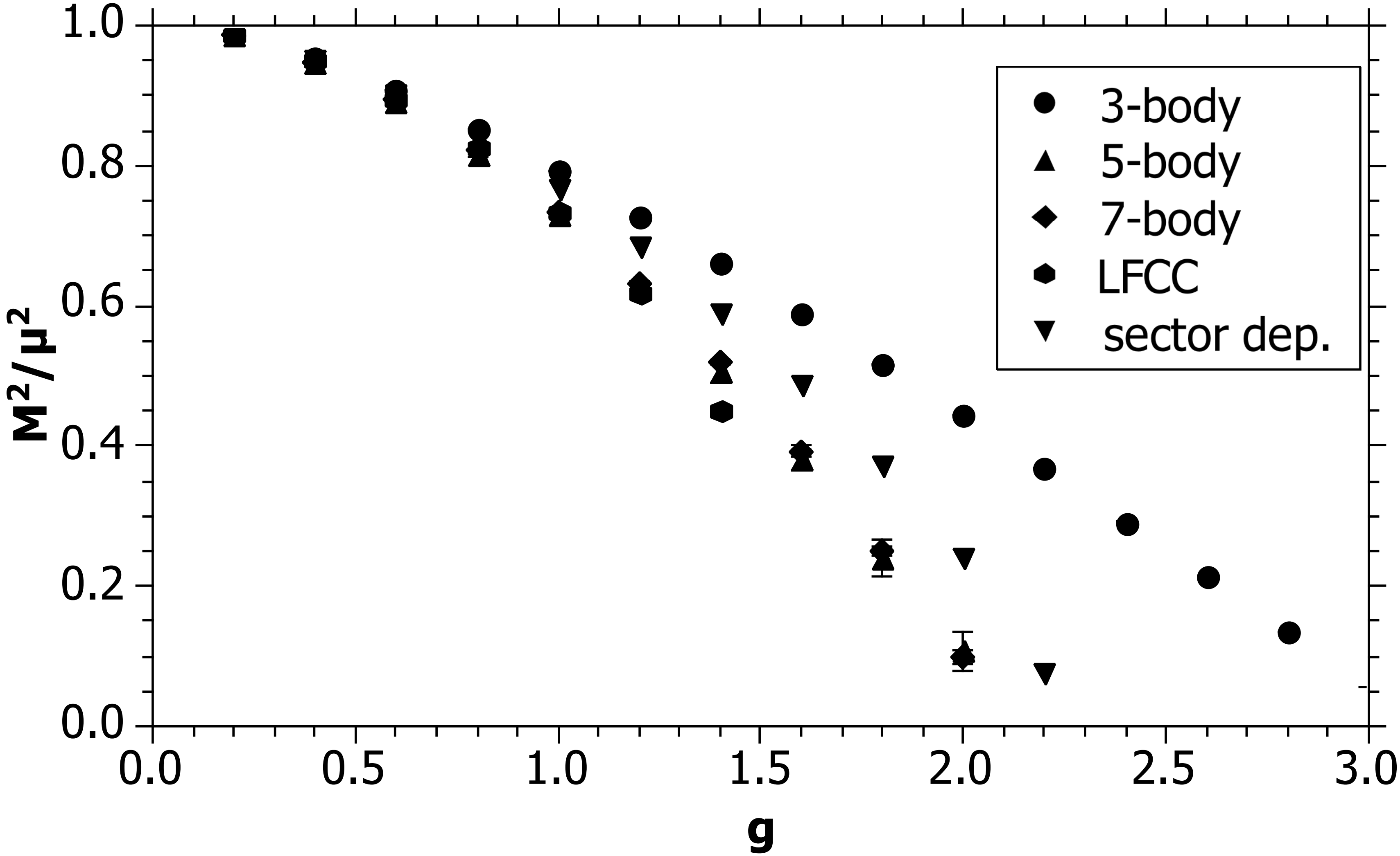
$$+\sqrt{g}\sum_{n'i'}V_{ni,n'i'}^{(m,m+1)}c_{n'i'}^{(m+1)}+\sqrt{g}\sum_{n'i'}V_{ni,n'i'}^{(m,m-1)}c_{n'i'}^{(m-1)}$$

$$=\frac{M^2}{\mu^2}\sum_{n'i'}B_{ni,n'i'}^{(m)}c_{n'i'}^{(m)}$$

Basis reduction

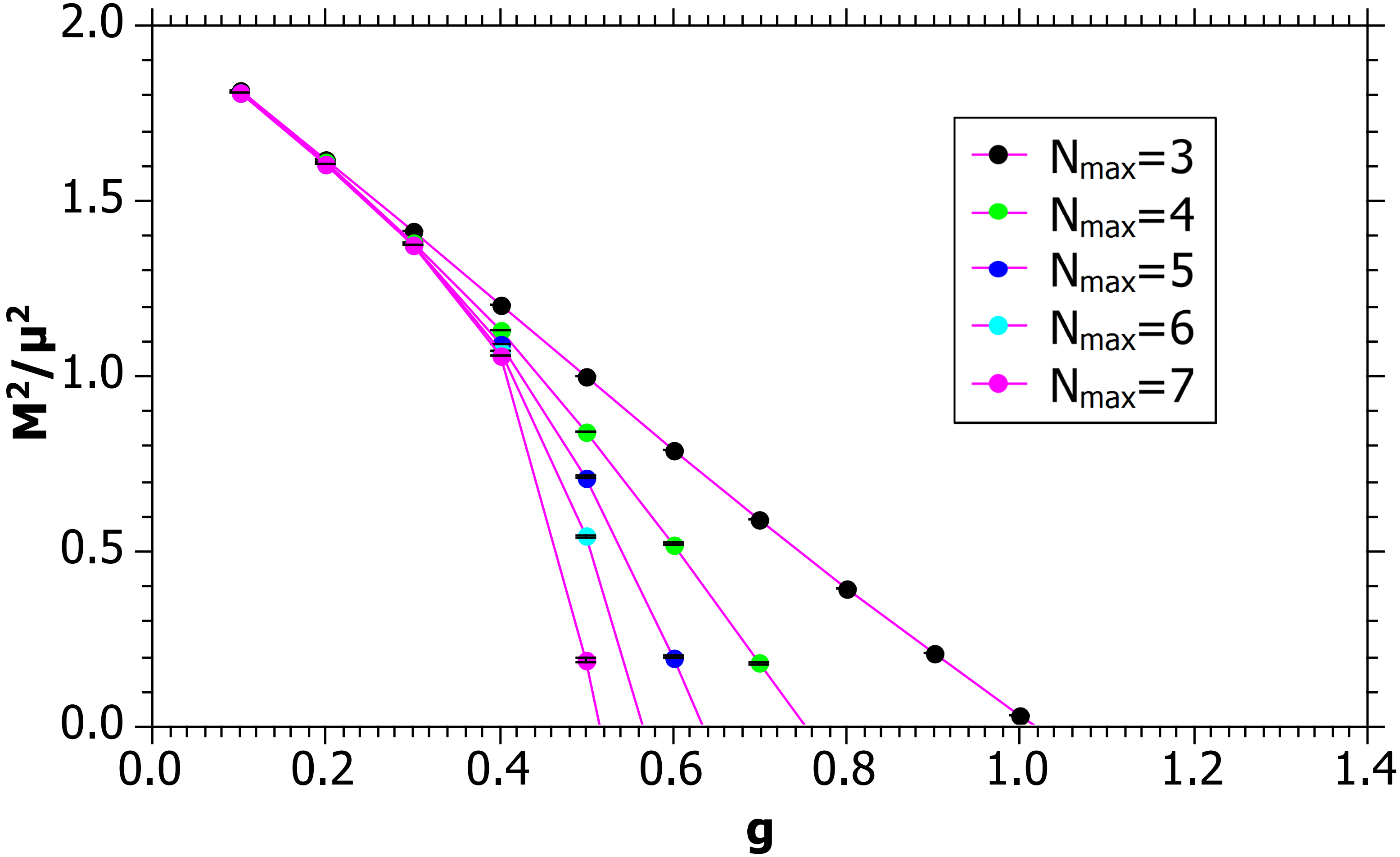
- $B^{(m)} =$ overlap of non-orthogonal basis functions in mth sector.
- implicitly orthogonalize by using a SVD $B^{(m)} = U^{(m)} D^{(m)} U^{(m)T}$.
  - columns of $U^{(m)}$ are eigenvectors of $B^{(m)}$.
  - $D^{(m)}$ is a diagonal matrix of the eigenvalues.
  - keep in $U^{(m)}$ only those columns associated with eigenvalues above some positive threshold.
    - [Wilson, NPB (Proc. Suppl.) **17**, 82 (1990)]
- define new vectors of coefficients $\vec{c}^{(m)\prime} = D^{1/2} U^T \vec{c}^{(m)}$ and new matrices, such as $T^{(m)\prime} = D^{-1/2} U^T T^{(m)} U D^{-1/2}$

Mass squared for unbroken phase



Critical coupling at $g = 2.1 \pm 0.05$.

Mass squared for asymmetric case



Each set of points corresponds to a different Fock-space truncation to $N_{\rm max}$ constituents.
Critical coupling values extrapolate to $g = 0.2 \pm 0.02$.

Chang's duality

[Chang, PRD **12**, 1071 (1975); **13**, 2778 (1976)]
[V.T. Kim *et al.*, PRD **69**, 085008 (2004)]
[A. Harindranath and J.P. Vary, PRD **37**, 3010 (1988)]

$$N_+[\phi^2] = N_-[\phi^2] + \frac{1}{4\pi}\ln\frac{\mu_+^2}{\mu_-^2},$$

$$N_+[\phi^4] = N_-[\phi^4] + 6\frac{1}{4\pi}\ln\frac{\mu_+^2}{\mu_-^2}N_-[\phi^2] + 3\left(\frac{1}{4\pi}\ln\frac{\mu_+^2}{\mu_-^2}\right)^2.$$

$$\mathcal{P}^- = \left(\frac{1}{2}\mu_+^2 + \frac{\lambda}{4}\frac{1}{4\pi}\ln\frac{\mu_+^2}{\mu_-^2}\right)N_-[\phi^2] + \frac{\lambda}{4!}N_-[\phi^4]$$
$$+\frac{1}{4\pi}\ln\frac{\mu_+^2}{\mu_-^2}\left(2\mu_+^2 + \frac{\lambda}{8}\frac{1}{4\pi}\ln\frac{\mu_+^2}{\mu_-^2}\right).$$

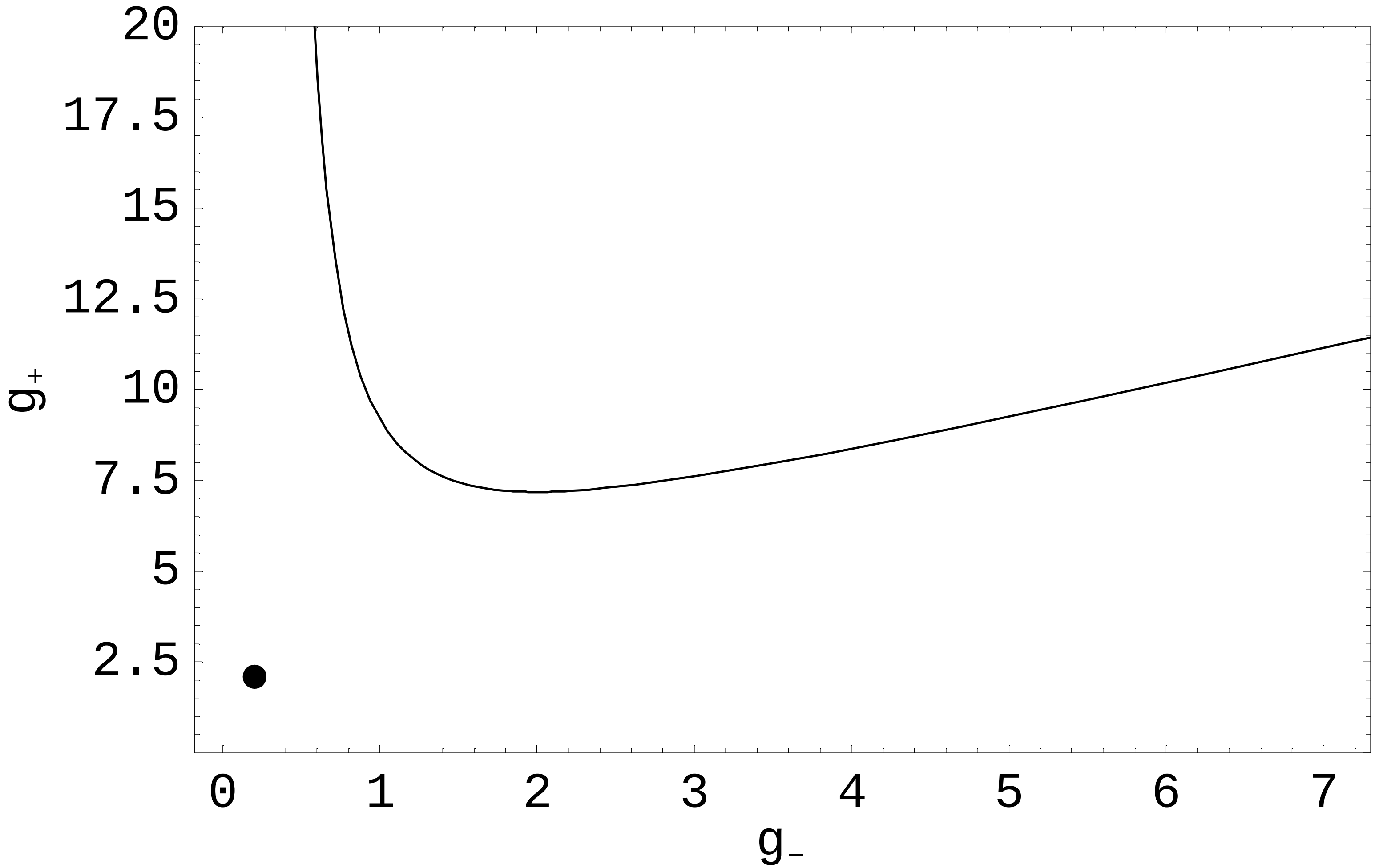
Equivalent to $\mathcal{P}^-$ with negative mass squared if

$$\frac{1}{2}\mu_+^2 + \frac{\lambda}{4}\frac{1}{4\pi}\ln\frac{\mu_+^2}{\mu_-^2} = -\frac{1}{2}\mu_-^2.$$

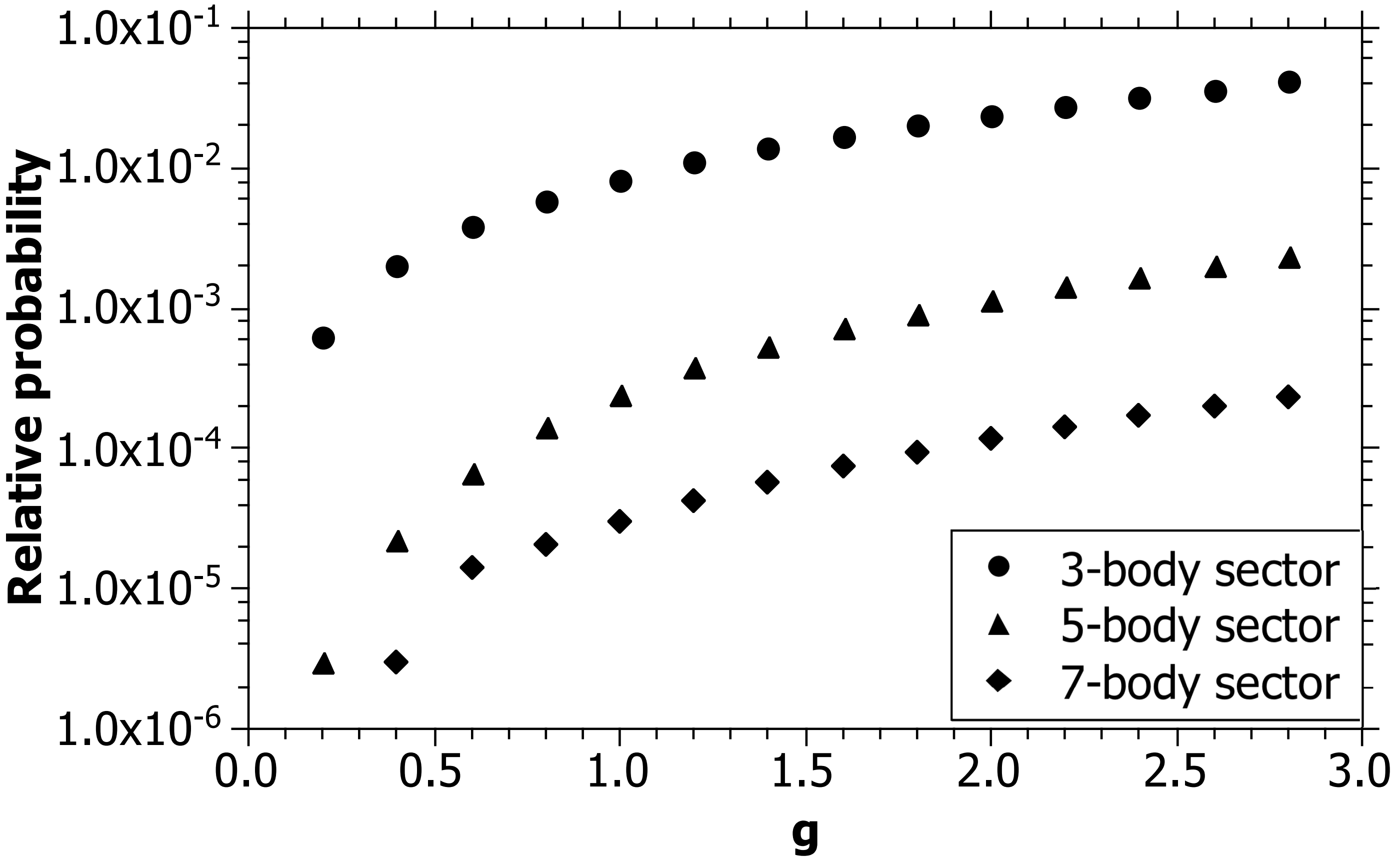
Duality for couplings

For the dimensionless couplings $g_\pm \equiv \lambda/4\pi\mu_\pm^2$

$$\frac{1}{g_+} - \frac{1}{2}\ln g_+ = -\frac{1}{2}\ln(g_-) - \frac{1}{g_-}.$$



Relative probabilities



No indication of critical behavior at $g = 2.1$.
Assume higher sectors suppressed by ‘large’ mass.
Use sector dependent mass μ_m.

Sector-dependent mass

Invariant mass in N-body sector: $\sum_i^N \frac{\mu^2}{1/N} = N^2\mu^2$

$$\sum_{n'i'}\left[\tilde\mu_m^2 T_{ni,n'i'}^{(m)\prime} \quad + \quad V_{ni,n'i'}^{(m,m)\prime}\right] c_{n'i'}^{(m)\prime} + \sum_{n'i'} V_{ni,n'i'}^{(m,m+2)\prime} c_{n'i'}^{(m+2)\prime}$$
$$+ \quad \sum_{n'i'} V_{ni,n'i'}^{(m,m-2)\prime} c_{n'i'}^{(m-2)\prime} = \tilde M^2 c_{ni}^{(m)\prime},$$

with $\tilde\mu_m \equiv \mu_m\sqrt{4\pi/\lambda}$ and $\tilde M \equiv M\sqrt{4\pi/\lambda}$.

$N_{\rm max} \to \infty \;\Rightarrow\; \tilde\mu_m^2 \to \tilde\mu \equiv \pm 4\pi\mu^2/\lambda$

$g \equiv \frac{\lambda}{4\pi\mu^2} \;\Rightarrow\; g \simeq 1/|\tilde\mu_1^2|$

and $M^2/\mu^2 = g\tilde M^2 = \tilde M^2/|\tilde\mu_1^2|$.

Solution for $\tilde{\mu}_1$

Define set of matrices $G^{(m)}$ recursively, from $m = N_{\rm max}$ down to 3, as

$$G^{(m)} = \left[\tilde{\mu}_m^2 T^{(m)\prime} + V^{(m,m)\prime} - \tilde{M}^2 I^{(m)} - V^{(m,m+2)\prime} G^{(m+2)} V^{(m+2,m)\prime}\right]^{-1},$$

with the initial form given by

$$G^{(N_{\rm max})} = \left[\tilde{M}^2 T^{(N_{\rm max})\prime} + V^{(N_{\rm max},N_{\rm max})\prime} - \tilde{M}^2 I^{(N_{\rm max})}\right]^{-1}$$

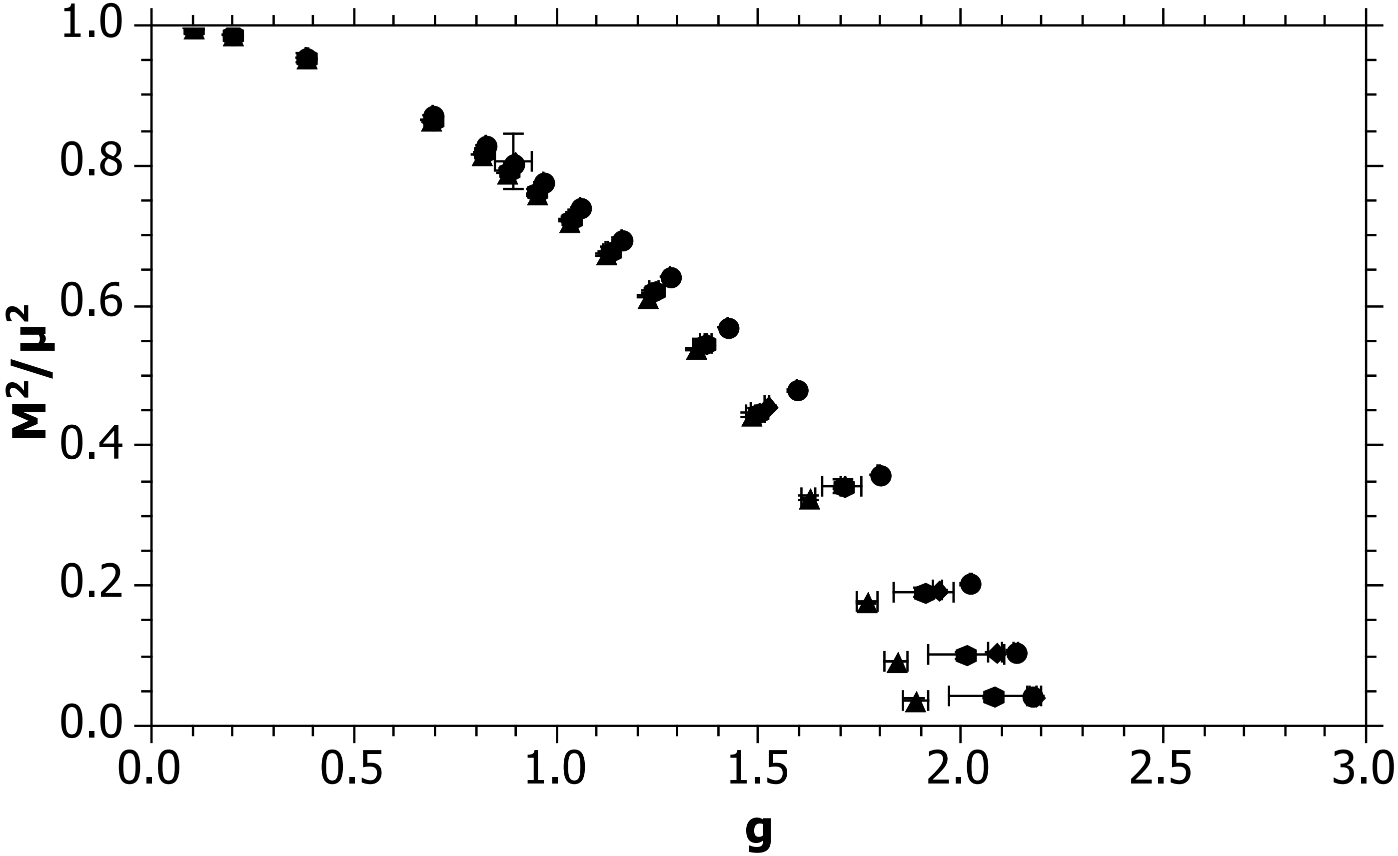
The mass in the lowest sector is then simply

$$\tilde{\mu}_1^2 = \frac{1}{T^{(1)}}\left[\tilde{M}^2 - V^{(1,1)\prime} - V^{(1,3)\prime} G^{(3)} V^{(3,1)\prime}\right].$$

Coefficients for the wave-function expansions are constructed recursively from $m = 3$ up to $N_{\rm max}$ by

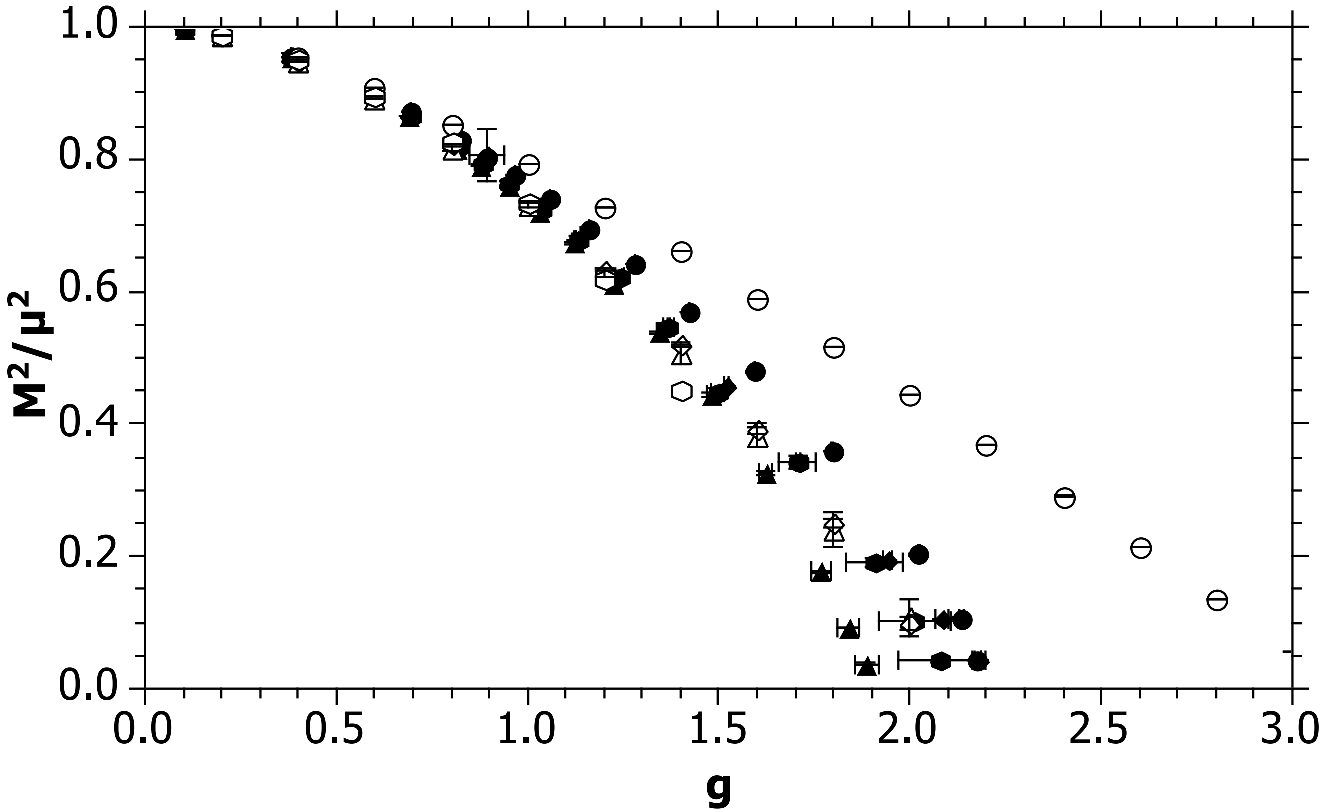
$$\vec{c}^{(m)\prime}/c^{(1)} = G^{(m)} V^{(m,m-2)\prime} \vec{c}^{(m-2)\prime}/c^{(1)}.$$

Results with sector dependence



$N_{\max} = 3$ (circles), 5 (triangles), 7 (diamonds), 9 (hex).
Error bars estimate the range of fits for μ_1 extrapolations.
Estimate critical coupling to be 2.1.

Combined results



Standard parameterization: open symbols up to $N_{\max} = 7$.
Five and seven-constituent results are nearly identical with the nine-constituent sector-dependent results.
LFCC results: open hexagons

Comparison

- results consistent with those from the standard parameterization
- the critical coupling is again estimated to be 2.1
- sector-dependent results do converge more slowly
  - they require $N_{\rm max} = 9$ compared to the $N_{\rm max} = 5$ for the standard parameterization.
  - to be expected, even desired, because we expect that higher Fock states should become more important as the critical coupling is approached.
  - but, how important?

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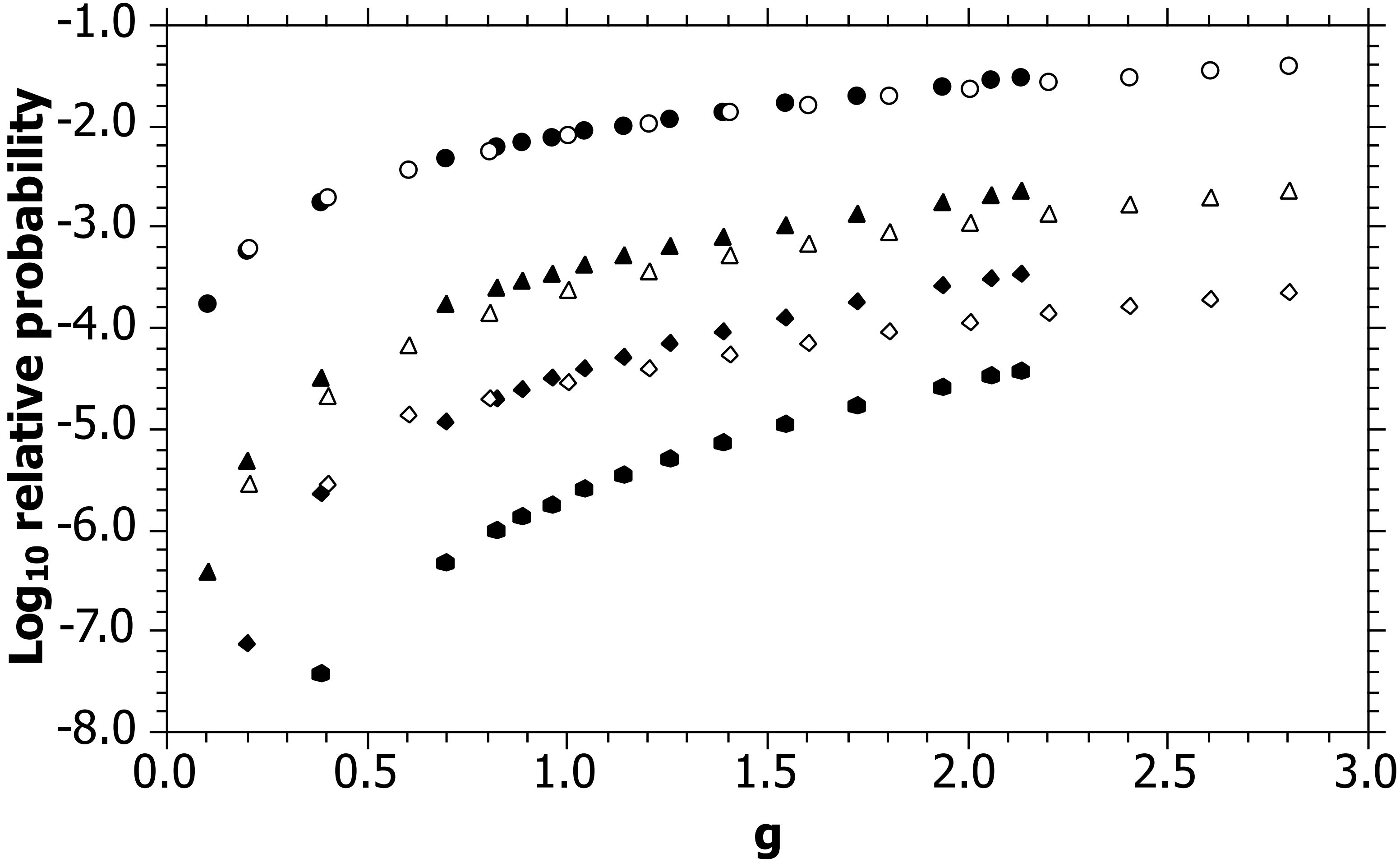
Sector-dependent relative probabilities



Computed with a truncation of $N_{\max} = 9$.
Open symbols for standard parameterization ($N_{\max} = 7$).

Conclusions

- the relative probabilities are essentially the same in the three-body Fock sector
  - indicating full convergence with respect to the Fock-space truncation.
- In Fock sectors with five and seven constituents, the relative probability for the sector-dependent case rises above the probability in the standard case as the critical coupling is approached.
  - the greater probability is expected
  - however, the full expectation was that these probabilities would increase more rapidly
- the original hypothesis, that sector-dependent masses would resolve the critical behavior, must be incorrect.

Summary

- need to understand calculation of the vev
  - in the presented solution for the broken phase, the vev remains nonzero at the (alleged) critical coupling
  - similarly, for the unbroken phase, the vev remains zero as the critical coupling is exceeded
  - a GEP analysis can determine a vev that does flip between zero and non-zero, but discontinuously
    - transition known to be not first order
- need to understand the convergence of the Fock space truncation near the critical coupling
  - more in next talk