Phase transition in Topologically Massive QED_3

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Abstract

Topologically Massive QED in (2+1) at T=0 is similar to XY model with finite chemical potential of vortex at finite-temperature and exhbits Kosterlitz-Thouless type transition.PTEP Febrary(2015)

Contents

0.1 Kosterlitz-Thouless transition (1973)

(2+1)-d boson model ; condensed phase

low temperature ;vortex pair are nuetral and there exists condensate

ϕ;phase fluctuation(continuous gauge transformation) ρ;uniform super fluid density,

$$\begin{aligned}\psi(x) &= \sqrt{\rho}e^{i\phi(x)},\\ \left\langle \psi^{+}(x)\psi(0)\right\rangle &= \rho\left\langle e^{-i\phi(x)}e^{i\phi(0)}\right\rangle\\ &= \rho e^{-(D(0)-D(x-y))}\\ &= \rho\exp(-\frac{k_BT}{2\pi K_0}\ln(\frac{r}{a})) \propto r^{-\eta}.\end{aligned}$$

where we used

$$\begin{aligned}&\left\langle 0|T(e^{-i\phi(x)}e^{i\phi(y)})|0\right\rangle\\ &= -\frac{1}{2}\left\langle 0|(\delta\phi^2(x)+\delta\phi^2(y)-2T(\delta\phi(x)\delta\phi(y))|0\right\rangle\\ &= -(D(0)-D(x-y)).\end{aligned}$$

a;short distance cut-off,$K_0 = (h/2\pi m)^2\rho d, d = thickness.$

vortex;singular gauge transformation (discontinuous)

$$\phi(r) = \phi(z) = \arctan(y/x) \qquad (1)$$

is a real angle in 2-dimension,satisfy $\Delta_x \phi(r) = \delta(x-y)$.

high temperature;single vortex excitation lower the energy but unstable.

Free energy may be written

$$F_\phi = \frac{1}{2}\int d^2 r K_0 |\nabla\phi|^2 = E_C + \pi K_0 \ln(\frac{L}{a})$$

$$F_{SV} = E - TS = \pi K_0 \ln(\frac{L}{a}) - k_B T \ln(\frac{L}{a})^2. \quad (2)$$

$$T \geq \frac{\pi K_0}{2k_B}, \quad (3)$$

free vortex may appear.From this fact critical temperature of super fluidity is determined

$T_c = \pi K_0/2k_B, \eta \geq 1/4$,high temperature phase $\rho = 0$. Dynamics ???????

0.2 Topologically Massive QED_3 with 4-component fermion

S.Deser, R.Jackiw and S.Templeton(1982)

$$\mathcal{L} = \mathcal{L}_{QED} - \frac{\mu}{4}\epsilon^{\mu\nu\rho}A_\mu F_{\mu\nu},$$

Maxwell equation $\nabla \cdot \mathbf{E} - \mu B = \rho$ has solution

$$\mathbf{A}(x)_{|x|\to\infty} \to \frac{-Q}{2\pi\mu}\nabla \arctan(\frac{y}{x}). \qquad (4)$$

There is a 2-spin degree of freedom.($\mu < 0, \mu > 0$):neutral parity conserving theory.(low temperature phase)

one degree:chiral (parity violating) theory,Topologically Massive QED.(high temperature phase)

Vortex is a degree of freedom of singular gauge transformation

$$\triangle_x \phi(x) = \delta^{(2)}(x), \phi(x) = \mu\pi \arctan(y/x) \qquad (5)$$

$\psi(r) \to \exp(i\phi(x))\psi(r)$,$\mu$ is related to Hall conductance.

0.3 symmetry

If the fermion is massless $m = 0$,$\mathcal{L}$ has $U(2)$ symmetry generated by $\{I, \gamma_3, \gamma_5, \tau\}$, $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$ for $\mu, \nu = (0, 1, 2)$, $\gamma_0 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}$, $\gamma_{1,2} = \begin{pmatrix} i\sigma_{1,2} & 0 \\ 0 & -i\sigma_{1,2} \end{pmatrix}$,

$\gamma_3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$, $\gamma_5 = \begin{pmatrix} 0 & -iI \\ iI & 0 \end{pmatrix}$,

$\tau = -i[\gamma_3, \gamma_5]/2 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$.γ_3 and γ_5 act as chiral transfomation

$$\begin{aligned} \psi(x) &\to e^{i\gamma_3\theta_3}\psi(x), \\ \psi(x) &\to e^{i\gamma_5\theta_5}\psi(x). \end{aligned} \tag{6}$$

Scalar density is mixed by chiral transformation

$$\begin{aligned} \overline{\psi}(x)\psi(x) \to \cos(2\theta_{3,5})\overline{\psi}(x)\psi(x) \\ + i\sin(2\theta_{3,5})\overline{\psi}(x)\gamma_{3,5}\psi(x), \end{aligned} \tag{7}$$

$$\overline{\psi}(x)\tau\psi(x) \to \overline{\psi}(x)\tau\psi(x). \tag{8}$$

Dynamical mass generation breaks $U(2)$ symmetry down to $U(1)_I \times U(1)_\tau$.

we have Ward-Takahashi relation

$$(p-q)_\mu \Gamma_\mu^{3,5}(p,q) = 2m_0 \Gamma^{3,5}(p,q) + \gamma_{3,5} S_F^{-1}(p) + S_F^{-1}(q)\gamma_{3,5}. \quad (9)$$

$$\lim_{p \longrightarrow q} (p-q)_\mu \Gamma_\mu^{3,5}(p,q) = \{\gamma_{3,5}, S_F^{-1}(p)\} \neq 0. \quad (10)$$

We find the eigenvalue of the free particle Hamiltonian

$$H = \gamma^0(\gamma^i p^i + m_e I + m_o \tau) \quad (11)$$

as $E^2 = p^2 + m_\pm^2, m_\pm = m_e \pm m_o$,where $i = 1, 2, I$ is a 4×4 unit matrix and τ is a operator defined above.Two kinds of mass are written by

$$m_e I + m_o \tau = \begin{pmatrix} m_+ & 0 \\ 0 & m_- \end{pmatrix} = \begin{pmatrix} m_e + m_o & 0 \\ 0 & m_e - m_o \end{pmatrix}. \quad (12)$$

So that we may split 4-component spinor into upper and lower components by projection operator

$$\psi_\pm = \chi_\pm \psi = \chi_\pm \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}, \chi_\pm = \frac{1 \pm \tau}{2}, \tag{13}$$

$$\chi_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \chi_- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{14}$$

Fom the Lagrangian

$$\mathcal{L} = \overline{\psi}_+(i\gamma \cdot \partial - m_+)\psi_+ + \overline{\psi}_-(i\gamma \cdot \partial - m_-)\psi_- \tag{15}$$

we have the free propagator

$$S(p) = -\frac{\gamma \cdot p + m_+}{p^2 - m_+^2}\chi_+ - \frac{\gamma \cdot p + m_-}{p^2 - m_-^2}\chi_-. \tag{16}$$

0.4 Dyson-Schwinger eqution

∘ Chiral symmetry breaking order parameter $\left\langle \overline{\psi}\psi \right\rangle = -Tr(S_F(x)) \to 0$ by vortex effects at critical μ_{cr}.

(1996) K.I.Kondo&P.Maris :1/N expansion $m_e = 0$,only m_o is produced.

(2011) A.Raya et.al vare vertex Landau gauge.both m_e&m_o are produced.$m_e \to 0$ first order phase transition occurs.

○ our works

$\mu_{cr} = 10^{-2}e^2$ by queched Landau gauge & full vertex correction with Ball-Chiu anzats.

$$(D^{-1})_{\mu\nu} = i(p^2 g_{\mu\nu} - p_\mu p_\nu + i\mu\epsilon_{\mu\nu\alpha}p^\alpha) + i\frac{p_\mu p_\nu}{\xi}. \tag{17}$$

$$(D^{-1})_{\mu\nu}D_{\nu\rho} = g_{\mu\rho}, \tag{18}$$

$$D_{\mu\nu}(k) = \frac{1}{i}[\frac{g_{\mu\nu} - k_\mu k_\nu/(k^2+i\epsilon) - i\mu\epsilon_{\mu\nu\rho}k^\rho/(k^2+i\epsilon)}{k^2-\mu^2+i\epsilon} + \xi\frac{k_\mu k_\nu}{(k^2+i\epsilon)^2}]. \tag{19}$$

The Schwinger-Dyson equation for the self-energy $\Sigma(p)$ for BC vertex is written

$$-i\Sigma(p) = (-ie)^2 \int \frac{d^3q}{(2\pi)^3} \frac{\gamma_\mu(q\cdot\gamma A(q) + B(q))}{(q^2A^2(q) - B^2(q))} \times \Gamma_\nu^{BC}(q,p) D_{\mu\nu}(q-p), \tag{20}$$

$$\begin{aligned}\Gamma_\mu^{BC}(p,q) &= \Gamma_\mu^T(p,q) + \frac{A(p)+A(q)}{2}\gamma_\mu \\ &+ \frac{A(p)-A(q)}{2(p^2-q^2)}\gamma\cdot(p+q)(p+q)_\mu \\ &- \frac{B(p)-B(q)}{p^2-q^2}(p+q)_\mu. \end{aligned} \tag{21}$$

W-T

$$(p-q)_\mu \Gamma_\mu^{BC}(p,q) = A(p)\gamma\cdot p - A(q)\gamma\cdot q - (B(p)-B(q)). \tag{22}$$

For bare vertex case

$$B(p)_\pm = \frac{e^2}{4\pi^2}\int_0^\infty \frac{dqq^2[B(q)_\pm I_0[p,q] \mp \mu A(q)_\pm I_2(p,q)_-]}{q^2A(q)_\pm^2 + B(q)_\pm^2},$$

$$A(p)_{\pm} = 1 + \frac{e^2}{4\pi^2 p^2} \times \int_0^{\infty} \frac{dqq^2[\pm\mu B(q)_{\pm} I_2(p,q)_{+} + A(q)_{\pm} I_3(p,q)]}{q^2 A(q)_{\pm}^2 + B(q)_{\pm}^2}.$$

For small $\mu(m_o)$,$m_+ > 0$,at $\mu = \mu_{cr}, m_+ = 0, B_+$ changes its sign.$\mu > \mu_{cr}, m_+ < 0$.

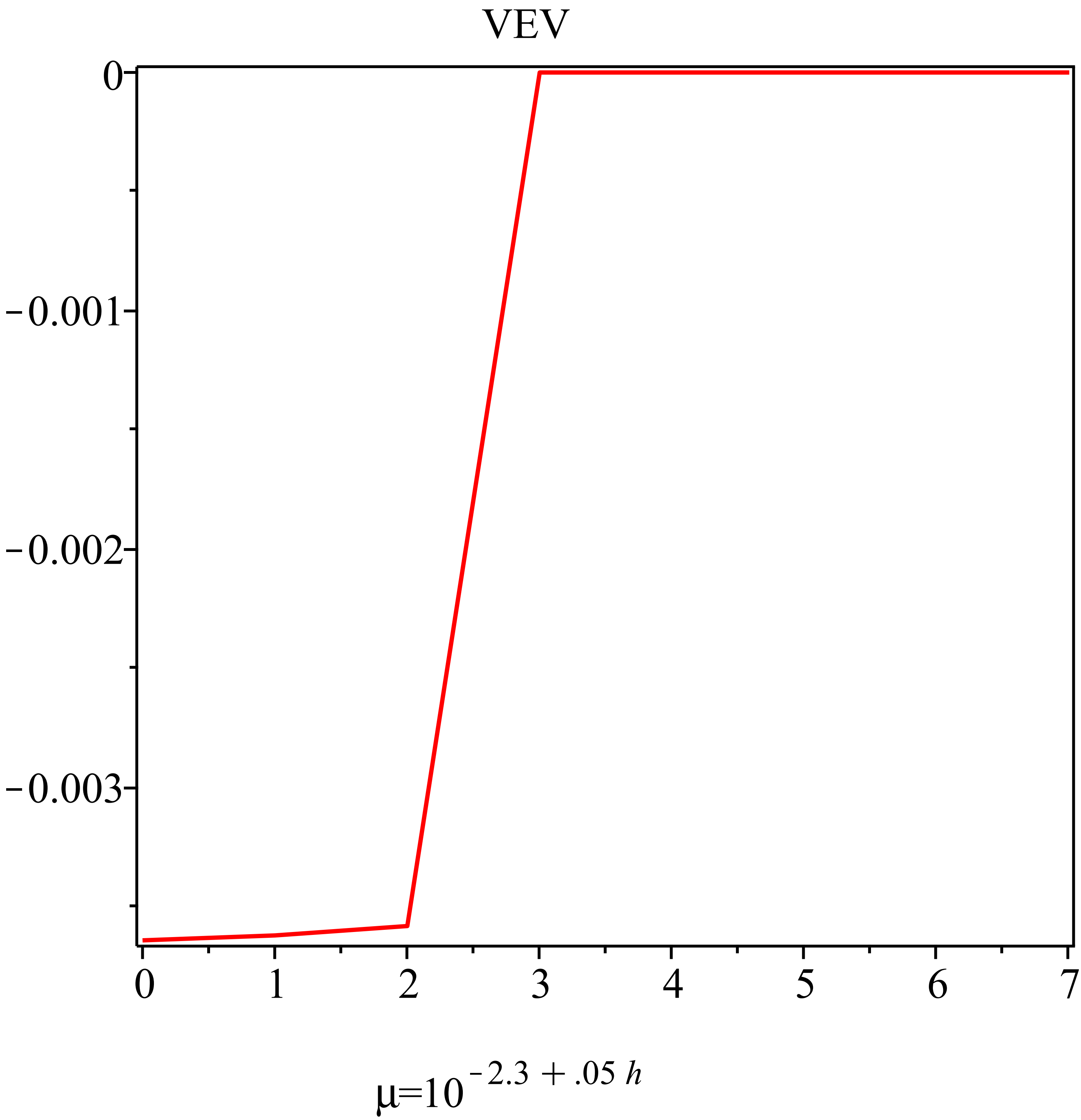
for $\mu = 0$

$$-\left\langle \overline{\psi}\psi \right\rangle = tr(S'_F) = \frac{e^4 e^{\gamma}}{32\pi^3} \sim 1.8 \cdot 10^{-3} e^4. \qquad (23)$$

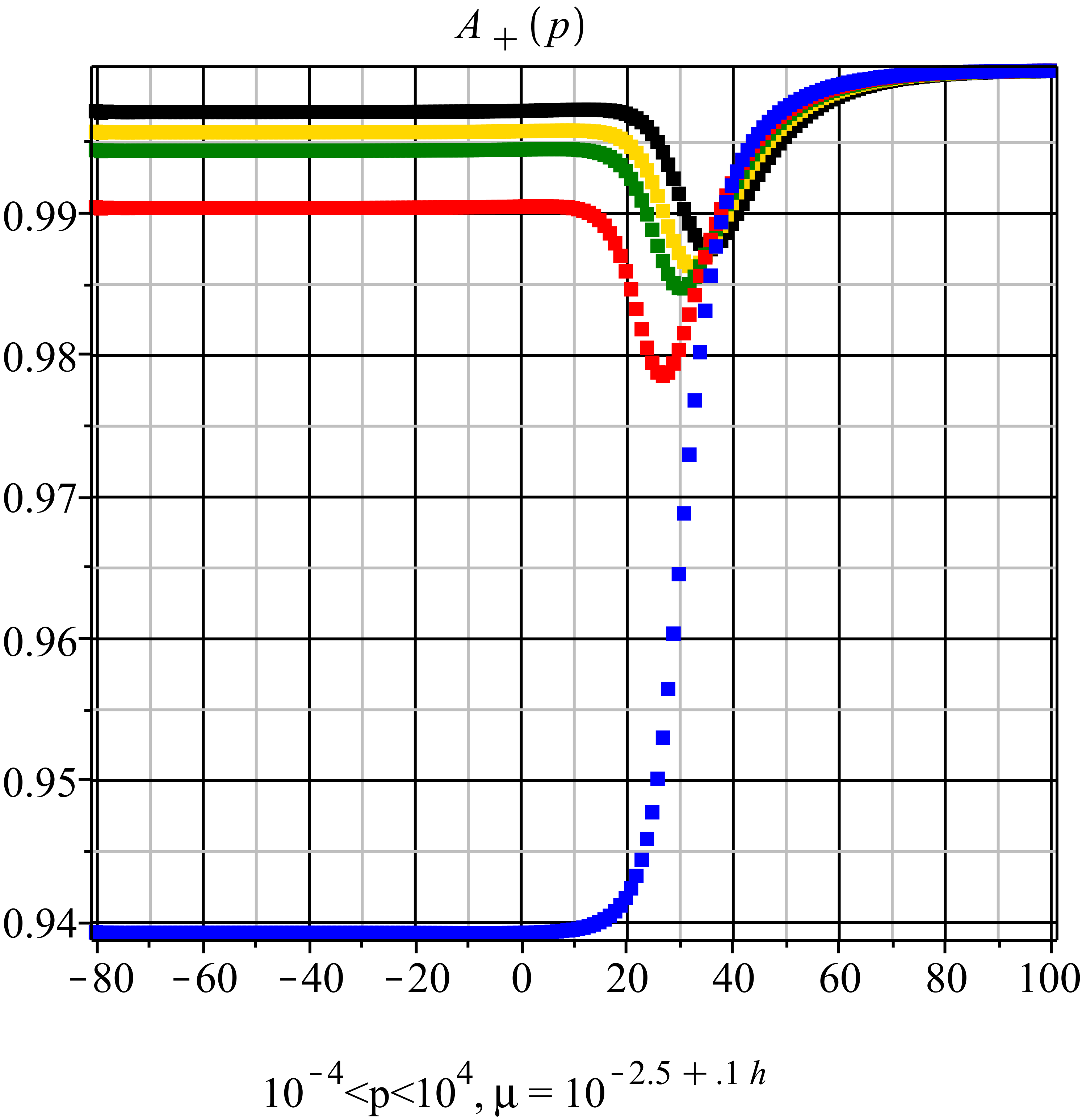
For $\mu \neq 0$ case ,we have two order parameter for parity even and odd

$$\left\langle \overline{\psi}\psi \right\rangle = \left\langle \overline{\psi}\psi \right\rangle_{+} + \left\langle \overline{\psi}\psi \right\rangle_{-}$$
$$\left\langle \overline{\psi}\tau\psi \right\rangle = \left\langle \overline{\psi}\psi \right\rangle_{+} - \left\langle \overline{\psi}\psi \right\rangle_{-}$$

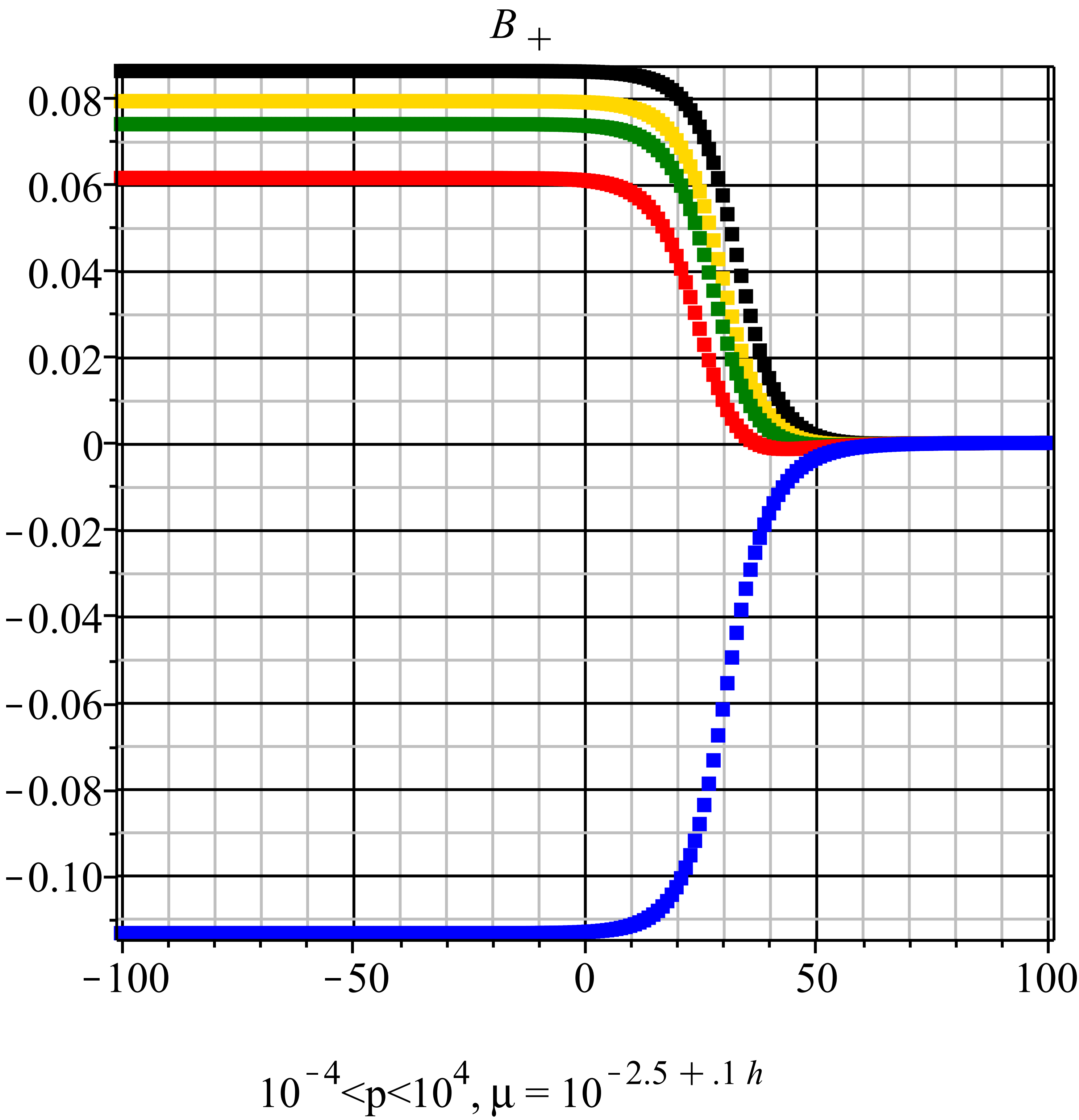
$$\mu_{cr} = 8 \times 10^{-3} e^2.$$



VEV
0
−0.001
−0.002
−0.003
0
1
2
3
4
5
6
7
μ=10^{−2.3 + .05 h}



$A_+(p)$
0.99
0.98
0.97
0.96
0.95
0.94
−80
−60
−40
−20
0
20
40
60
80
100
$10^{-4} < p < 10^4, \mu = 10^{-2.5 + .1\,h}$


B_+
0.08
0.06
0.04
0.02
0
−0.02
−0.04
−0.06
−0.08
−0.10
−100
−50
0
50
100
$10^{-4}<p<10^{4}, \mu = 10^{-2.5+.1h}$

0.5 Summary

We studied the effects of vortex on the chiral condensate by Dyson-Schwinger equation of the self-energy of fermion in Topologically Massive QED_3.These show the detaild dynamics of vortex on condensed matter physics which has not been clear.

K.I.Kondo &P.Maris(1996),A.Raya et.al(2011),

Y.Hoshino,T.Inagaki,Y.Mizutani(2015)

argued 1-st order phase transition.Physical meanings has not been clear.

Finite Temperature case is now in progress.