# Alternate light front quantization procedure for scalar fields

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- Dirac's method of quantization for constrained systems give (Dirac) brackets for all fields
- Lorentz symmetry may lead to singular commutator at LF
- one should quantize constrained systems at LF without Dirac's method

# Outlook

1. Standard light front quantization
   - Definitions in D = 3+1
   - Standard LF procedure
2. Novel light front quantization procedure
   - Translation symmetry
   - Novel LF assumptions
3. Dipole scalar field
   - light front Schrödinger equations
   - LF Feynman propagator
4. Conclusions and prospects

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coordinates

$$x^+ = \frac{x^0 + x^3}{\sqrt{2}}, \quad x^- = \frac{x^0 - x^3}{\sqrt{2}}, \quad \boldsymbol{x}_\perp = (x^1, x^2)$$

momenta

$$p^+ = \frac{p^0 + p^3}{\sqrt{2}}, \quad p^- = \frac{p^0 - p^3}{\sqrt{2}}, \quad \boldsymbol{p}_\perp = (p^1, p^2)$$

Minkowski space metric tensor

$$\eta_{\mu\nu} = g_{\mu\nu} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

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partial derivatives

$$\partial_+ = \frac{\partial}{\partial x^+}, \quad \partial_- = \frac{\partial}{\partial x^-}, \quad \partial_i = \frac{\partial}{\partial x^i}, \quad i \in \{1, 2\}$$

light front momenta and coordinates (at $x^+ = 0$)

$$\bar{k} = \left(k^+ \in (0, \infty), \boldsymbol{k}_\perp \in (-\infty, \infty)\right), \qquad \bar{x} = \left(x^- \in (-\infty, \infty), \boldsymbol{x}_\perp \in (-\infty, \infty)\right)$$

restricted scalar product

$$\bar{k} \cdot \bar{x} = k^+ x^- - \boldsymbol{k}_\perp \cdot \boldsymbol{x}_\perp$$

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# Standard procedure for scalar field

following *An introduction to light-front dynamics for pedestriants*, by A.Harindranath (1996) J.P. Vary and F. Woelz (eds.)

- Lagrangian density

$$\mathcal{L} = \partial_+ \phi \partial_- \phi - \frac{1}{2}(\partial_i \phi)^2 - \frac{m^2}{2}\phi^2$$

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- translation generators

$$P_\mu=:\int_{\mathbb{R}^3}\mathrm{d}^3\bar{x}\,T^+_{\ \mu}(\bar{x})\ :$$

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# LF assumption I and canonical commutator

- LF assumption I at $x^+ = 0$ the infinitesimal translation equation

$$\mathrm{i}\partial_-\phi(0,\bar{x}) = [\phi(0,\bar{x}), P_-] \,,$$

where the energy-momentum component $T^{++} = (\partial_-\phi)^2$ gives

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- the vacuum expectation value gives

$$\langle 0|\phi(0,\bar{x})\,\partial_-\phi(0,\bar{y})|0\rangle - \langle 0|\partial_-\phi(0,\bar{y})\,\phi(0,\bar{x})|0\rangle = \frac{\mathrm{i}}{2}\delta^3(\bar{x}-\bar{y})$$

with normalized vacuum state $\langle 0|0\rangle = 1$.

# LF assumption II and decomposition of Wightman functions

- we need to decompose the equation on the LF hypersurface

$$\langle 0|\phi(0)\mathrm{e}^{-\mathrm{i}\bar{P}\cdot(\bar{x}-\bar{y})}P_-\,\phi(0)|0\rangle + \langle 0|\phi(0)\mathrm{e}^{\mathrm{i}\bar{P}\cdot(\bar{x}-\bar{y})}P_-\,\phi(0)|0\rangle = \frac{1}{2}\delta^3(\bar{x}-\bar{y}),$$

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- smooth scalar field at LF hypersurface with test function $f(\bar{x}) \in \mathscr{S}'(\mathbb{R}^3)$

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- where 1-particle state is an eigenstate of kinematic momenta and the Fourier transform

$$|\bar{k}\rangle = a^\dagger(\bar{k})|0\rangle, \quad P_-|\bar{k}\rangle = k^+|\bar{k}\rangle, \quad P_i|\bar{k}\rangle = k_i|\bar{k}\rangle, \quad \tilde{f}(\bar{k}) = \int_{\mathbb{R}^3} d^3\bar{x} f(\bar{x})\, e^{i\bar{k}\cdot\bar{x}}$$

# decomposition of smooth Wightman functions

- after integration with test function $f(\bar{x}-\bar{y})$ we obtain smooth relation

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- or equivalently the commutator for creation and annihilation operators

$$\left[a(\bar{k}), a^\dagger(\bar{p})\right] = \delta^\Gamma(\bar{k},\bar{p})$$

# Reconstruction of field

- light front Schrödinger equation for smooth field according to LF assumption II gives

$$
\begin{aligned}
0 &= \left(-2P_+P_- + P_\perp^2 + m^2\right)\phi[f]|0\rangle = \\
&= \int_{\mathbb{R}^3} \mathrm{d}\Gamma(\bar{k})\tilde{f}(\bar{k})\left[-2k^+P_+ + k_\perp^2 + m^2\right]|\bar{k}\rangle
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- this allows to evaluate temporal evolution for $\phi(x)$

$$\phi(x) = \boxed{e^{iP_+x^+}\phi(\bar{x})e^{-iP_+x^+}} = \int_{\mathbb{R}^3} \mathrm{d}\Gamma(\bar{k})\left[e^{-i\bar{k}\cdot\bar{x}}e^{-i\frac{k_\perp^2+m^2}{2k^+}x^+}a(\bar{k}) + e^{i\bar{k}\cdot\bar{x}}e^{i\frac{k_\perp^2+m^2}{2k^+}x^+}a^\dagger(\bar{k})\right].$$

# Reconstruction of translation generators

- commutator

$$\langle 0|a(\bar{k})a^{\dagger}(\bar{p})|0\rangle = \delta^{\Gamma}(\bar{k},\bar{p}), \qquad \left[a(\bar{k}), a^{\dagger}(\bar{p})\right] = \delta^{\Gamma}(\bar{k},\bar{p}).$$

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$$\begin{aligned} P_{+} = H_{LF} &= \int_{\mathbb{R}^3} d\Gamma(\bar{k})\, \frac{m^2+k_{\perp}^2}{2k^{+}}\, a^{\dagger}(\bar{k})a(\bar{k}), \\ P_{-} = P^{+} &= \int_{\mathbb{R}^3} d\Gamma(\bar{k})\, k^{+}\, a^{\dagger}(\bar{k})a(\bar{k}), \\ P_{i} &= \int_{\mathbb{R}^3} d\Gamma(\bar{k})\, k_{i}\, a^{\dagger}(\bar{k})a(\bar{k}). \end{aligned}$$

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# Dipole scalar field

- lagrangian density with two scalar fields $\phi$ and $\chi$

$$\mathcal{L} = \partial_+\phi\partial_-\chi + \partial_-\phi\partial_+\chi - \partial_i\phi\partial_i\chi + \frac{m^2}{2}\chi^2 - m^2\chi\phi,$$

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- LF assumption I leads to nonvanishing canonical commutator

$$2\left[\phi(\bar{x}), \partial_-\chi(0)\right] = i\delta(x^-)\delta^2(\boldsymbol{x}_\perp)$$

# Commutators and Wightman functions

- for decomposition of light front commutator function (VEV)

$$2\langle 0|\,[\phi(\bar{x}), \partial_- \chi(0)]\,|0\rangle = i\delta^3(\bar{x})$$

we use the unitary representation of quantum scalar fields

$$\phi(x) = e^{\mathrm{i}P\cdot x}\phi(0)e^{-\mathrm{i}P\cdot x}, \quad \chi(x) = e^{\mathrm{i}P\cdot x}\chi(0)e^{-\mathrm{i}P\cdot x},$$

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$$\boxed{\phi[f]|0\rangle = \int_{\mathbb{R}^3} \mathrm{d}\Gamma(\bar{k})\tilde{f}(\bar{k})|\bar{k}, a\rangle, \quad \chi[f]|0\rangle = \int_{\mathbb{R}^3} \mathrm{d}\Gamma(\bar{k})\tilde{f}(\bar{k})|\bar{k}, b\rangle,}$$

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$$\boxed{\langle 0|\phi(0)|\bar{k},a\rangle = 0, \quad \langle 0|\chi(0)|\bar{k},a\rangle = 1, \quad \langle 0|\phi(0)|\bar{k},b\rangle = 1, \quad \langle 0|\chi(0)|\bar{k},b\rangle = 0}$$

# Outlook

1. Standard light front quantization
   - Definitions in D = 3+1
   - Standard LF procedure

2. Novel light front quantization procedure
   - Translation symmetry
   - Novel LF assumptions

3. Dipole scalar field
   - light front Schrödinger equations
   - LF Feynman propagator

4. Conclusions and prospects

# light front Schrödinger equations

- equations of motion $(\Box + m^2)\phi = m^2\chi$, $(\Box + m^2)\chi = 0$ lead the light front Schrödinger equations

$$\boxed{\left(-2P_+P_- + P_\perp^2 + m^2\right)\phi(x)|0\rangle = m^2\chi(x)|0\rangle, \quad \left(-2P_+P_- + P_\perp^2 + m^2\right)\chi(x)|0\rangle = 0.}$$

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$$\left[P_+, b^\dagger(\bar{k})\right] = \frac{m^2 + k_\perp^2}{2k^+} b^\dagger(\bar{k}), \quad \left[P_+, a^\dagger(\bar{k})\right] = \frac{m^2 + k_\perp^2}{2k^+} a^\dagger(\bar{k}) - \frac{m^2}{2k^+} b^\dagger(\bar{k})$$

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- momentum representation of translation generators $P_\mu$

$$\begin{aligned}
P_+ = H_{LF} &= \int_{\mathbb{R}^3} d\Gamma(\bar{k}) \left[-\frac{m^2}{2k^+} b^\dagger(\bar{k})b(\bar{p}) + \frac{m^2 + k_\perp^2}{2k^+}\left(b^\dagger(\bar{k})a(\bar{k}) + a^\dagger(\bar{k})b(\bar{k})\right)\right], \\
P_- = P^+ &= \int_{\mathbb{R}^3} d\Gamma(\bar{k})\, k^+ \left[b^\dagger(\bar{k})a(\bar{k}) + a^\dagger(\bar{k})b(\bar{k})\right], \\
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# field operators and Wightman function

- Finally we obtain the momentum representation for scalar fields

$$\begin{aligned}\chi(x) &= \int_{\mathbb{R}^3} d\Gamma(\bar{k}) \left[ \mathrm{e}^{-\mathrm{i}\bar{k}\cdot\bar{x}} \mathrm{e}^{-\mathrm{i}(m^2+k_\perp^2)/(2k^+)x^+} b(\bar{k}) + h.c. \right], \\ \phi(x) &= \int_{\mathbb{R}^3} d\Gamma(\bar{k}) \left[ e^{-\mathrm{i}\bar{k}\cdot\bar{x}} \mathrm{e}^{-\mathrm{i}(m^2+k_\perp^2)/(2k^+)x^+} \left( a(\bar{k}) + \mathrm{i}\frac{m^2 x^+}{2k^+} b(\bar{k}) \right) + h.c. \right]\end{aligned}$$

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# LF Feynman propagator

- chronological product in $x^+$ variable gives

$$\begin{aligned}\langle 0|T^+\phi(x)\phi(0)|0\rangle &= \theta(x^+)\langle 0|\phi(x)\phi(0)|0\rangle + \theta(-x^+)\langle 0|\phi(0)\phi(x)|0\rangle = \\ &= \frac{im^2}{(2\pi)^3}\int_{\mathbb{R}^4} e^{-ik\cdot x}\boxed{\int_0^\infty \frac{d\lambda}{4\lambda^3}e^{i\frac{k^2-m^2}{2\lambda}}} = \\ &= -\frac{im^2}{(2\pi)^4}\int_{\mathbb{R}^4} e^{-ik\cdot x}\boxed{\frac{1}{(k^2-m^2+i0^+)^2}}\end{aligned}$$

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- Lorentz symmetry of LF Feynman propagator is explicit
- double pole appears naturally

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*Thank you for your attention*