In-Medium Pion Valence Distributions In a Light-Front Model

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Collaborations: J.P.B.C. de Melo, I. Ahmed, T. Frederico

References:

Pion (Kaon) in vacuum and medium:
J. P. B. C de Melo, K. Tsushima, I. Ahmed, arXiv:1608.03858
AIP Conf.Proc. 1735, 080012 (2016), arXiv:1512.07260
AIP Conf.Proc. 1735, 080006 (2016), arXiv:1511.09219
Phys.Rev. C90, no.3, 035201 (2014)

QMC Model:
K. Saito, K. Tsushima and A. W. Thomas,
Progress Part. Nucl. Phys. 58, 1 (2007)

Light-Front Motivations

• Poisson bracket relations are retained invariant (Dirac (1949)) also in Light-Front approach !! [dynamical variable transformations]

• Ligh-Front has advantages to Describe Hadronic Bound States

• Constituent Picture of the Hadronic System

• Light-Front Wave functions: Representation of Composite Systems in QFT

• Invariant Under Boosts

• Light-Front Vacuum is Trivial

• LF Lorentz Invariant Hamiltonian: $P^2 = P^+P^- - \vec{P}_\perp^2$

• In-Medium Extension !!! First Trial: Pion

(Later: Rho meson, Kaon, D-meson, Nucleon, etc. etc.)

Light-Front Coordinates

Four-Vector $\implies$ $x^\mu = (x^0, x^1, x^2, x^3) \rightarrow (x^+, x^-, \vec{x}_\perp)$

$x^+ = t + z$ $\quad x^+ = x^0 + x^3 \implies$ **Time**

$x^- = t - z$ $\quad x^- = x^0 - x^3 \implies$ **Position**

Metric Tensor and Scalar product

$$x \cdot y = x^\mu y_\mu = x^+ y_+ + x^- y_- + x^1 y_1 + x^2 y_2 = \frac{x^+ y^- + x^- y^+}{2} - \vec{x}_\perp \vec{y}_\perp$$

$p^+ = p^0 + p^3$, $p^- = p^0 - p^3$, $\vec{p}_\perp = (p^1, p^2)$

Dirac Matrix and Electromagnetic Current

$\gamma^+ = \gamma^0 + \gamma^3 \implies$ **Electr. Current** $J^+ = J^0 + J^3$

$\gamma^- = \gamma^0 - \gamma^3 \implies$ **Electr. Current** $J^- = J^0 - J^3$

$\gamma^\perp = (\gamma^1, \gamma^2) \implies$ **Electr. Current** $J^\perp = (J^1, J^2)$

$$p^\mu x_\mu = \frac{p^+ x^- + p^- x^+}{2} - \vec{p}_\perp \vec{x}_\perp$$

$$x^+, x^-, \vec{x_\perp} \implies p^+, p^-, \vec{p}_\perp$$

$p^- \implies$ **Light-Front Energy**

$$p^2 = p^+ p^- - (\vec{p}_\perp)^2 \implies \quad p^- = \frac{(\vec{p}_\perp)^2 + m^2}{p^+}$$

On-shell

Bosons $\implies \quad D(p) = \frac{1}{p^2 - m^2 + \imath\epsilon}$

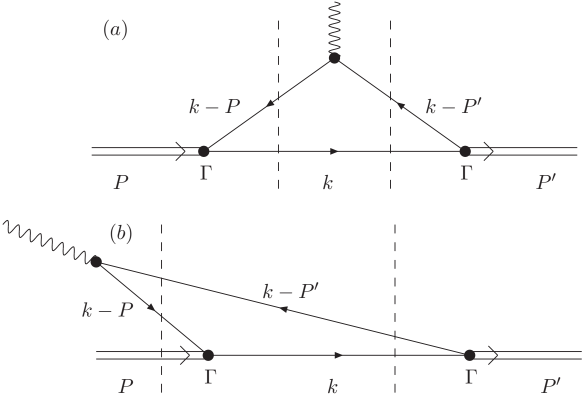
Fermions $\implies \quad S_F(p) = \frac{\not{p}_{on} + m}{p^+(p^- - p^-_{on} + \frac{\imath\epsilon}{p^+})} + \frac{\gamma^+}{2p^+}, \quad p^-_{on} = \frac{(\vec{p}_\perp)^2 + m^2}{p^+} > 0$

Review Papers:

- **Phys. Rept. 301, (1998) 299-486, Brodsky, Pauli and Pinsky**
- **A. Harindranath, Pramana, Journal of Indian Academy of Sciences Physics Vol.55, Nos 1 & 2, (2000) 241.**
- **An Introduction to Light-Front Dynamics for Pedestrians**

Avaroth Harindranath

Light-Front book organizers: James Vary and Frank Wolz (1997)



(a) ⇒ Valence Component of the Electromagnetic Current
(b) ⇒ Non-Valence Component of the Electromagnetic Current
Ref.: de Melo and Frederico, PRC (1997) , de Melo, Naus, Frederico and Sauer, PRC(1999)

Effective Lagrangian to Vertex $\pi \to q\bar{q}$

$$\mathcal{L}_I = -\imath \frac{m}{f_\pi} \vec{\pi} \cdot \overline{q} \gamma^5 \vec{\tau} q$$

- **Electromagnetic Current:** J_π^+

$$\begin{aligned} J^\mu &= -\imath 2e \frac{m^2}{f_\pi^2} N_c \int \frac{d^4 k}{(2\pi)^4} Tr[S(k)\gamma^5 \\ &\times S(k-P')\gamma^\mu S(k-P)\gamma^5 \Lambda(k,P')\Lambda(k,P)] \end{aligned}$$

$S(p) = \frac{1}{\not{p} - m + \imath\epsilon}$

- **Vertex Function**
- Symmetric Vertex Function

$$\Lambda(k,P) = \frac{N}{(k^2 - m_R^2 + \imath\epsilon)} + \frac{N}{((P-k)^2 - m_R^2 + \imath\epsilon)}$$

- Ref. Nucl. Phys. **A 707** (2002) 399-424
- Nonsymmetric Vertex Function

$$\Lambda(k,P) = \frac{N}{((P-k)^2 - m_R^2 + \imath\epsilon)}$$

- J.P.B.C. de Melo, T. Frederico and H.L. Naus,

Phy.Rev. **C59** (1999) 2278

- Frame

$$\begin{aligned} q^+ &= -q^- = \sqrt{-q^2}\sin\alpha \\ q_x &= \sqrt{-q^2}\cos\alpha,\ \ q_y = 0 \\ q^2 &= q^+q^- - (q_\perp)^2 \ . \end{aligned}$$

- Breit Frame $(\alpha = 0) \implies q^+ \to 0\ , q^- = 0\ ;\ \vec{q} \neq 0$
- $J^+_\pi = J^0 + J^3 \implies$ **No Pair Term Contribuition (Only Valence!!)**

In this frame, and spin 0 case true, NOT spin 1 !!!

- $J^-_\pi = J^0 - J^3 \implies$ **Pair Term Contribuition**
- **de Melo, Frederico, Pace and Salmé, NPA 707 (2002) 399**
- **de Melo, Frederico and Naus, PRC 59 (1999) 2278**

(Valence) Wave Function, Pion // Kaon

$$
\begin{aligned}
\Psi(x, k_\perp, p^+, \vec{p}_\perp) \;\propto\; & \left[\frac{1}{(1-x)(m_{0^-}^2 - \mathcal{M}^2(m_q^2, m_R^2))} \right. \\
+ \; & \left. \frac{1}{x(m_{0^-}^2 - \mathcal{M}^2(m_R^2, m_{\bar{q}}^2))}\right] \frac{1}{m_{0^-}^2 - \mathcal{M}^2(m_q^2, m_{\bar{q}}^2)}, \\
+ \; & [q \leftrightarrow \bar{q}]
\end{aligned}
$$

here:

$$
\mathcal{M}^2(m_a^2, m_b^2) = \frac{k_\perp^2 + m_a^2}{x} + \frac{(p-k)_\perp^2 + m_b^2}{(1-x)} - p_\perp^2
$$

- **Quark mass** $\Rightarrow$ **Free Mass Operador**
- m_{0^-} $\Longleftrightarrow$ **Mass of the bound state**

• **Motivation: The present model works well (Symmetric Vertex)!**

Observables: Decay constant and charge radius						
	f_{0^-} (MeV)	r_{0^-}	m_u (π^+)	m_d (π^+)	m_d (K^+)	$m_{\bar{s}}$ (K^+)
Pion	**93.12** **101.85**	**0.736** **0.670**	**220** **250**	**220** **250**		
Kaon	**101.81** **113.74**	**0.754** **0.687**			**220** **250**	**440** **440**

$m_R = 600\ MeV$, **(all masses in MeV and radius in fm)**
Ex.(Pion): $f_\pi = 92.4 \pm 0.021\ MeV$, $r_\pi = 0.672 \pm 0.08\ fm$ **(PDG)**
Ex.(Kaon): $f_{k^+} = 110.38 \pm 0.1413\ MeV$, $r_{k^+} = 0.560 \pm 0.031$ **(PDG)**

• **Ref.: de Melo, Frederico, Pace and Salmè, NPA707, 399 (2002); ibid., Braz. J. Phys. 33, 301 (2003)**
•**Yabusaki, Ahmed, Paracha, de Melo, El-Bennich, PRD92 (2015) 034017.**

QMC (Quark-Meson Coupling)⋆ "plus" Light-Front

- **Hatree Mean Field Approximation:** $\Rightarrow \quad p^\mu \longrightarrow p^\mu + V^\mu$

- **Vector**

$$p^\mu \pm \delta_0^\mu V_\omega^q = \begin{cases} + \;, & \textbf{quark} \\ - \;, & \textbf{antiquark} \end{cases}$$

- **Scalar:** V_s

$\Rightarrow \quad m_q \longrightarrow m_q^* + V_s$ **here** $V_s = m_q - V_\sigma^q$

⋆ **QMC Ref.: K. Saito, K. Tsushima and A. W. Thomas, Progress Part. Nucl. Phys. 58, 1 (2007)**

- **Propagadors of Quarks in Mediun**

$$S_F^*(p+V) = \frac{1}{\not p - \not V - m_q^* + \imath\epsilon}$$

- **Vertex $q\pi\bar{q}$ in Medium**

$$\Lambda^*(k+V,P) = \frac{C^*}{((k+V)^2 - m_R^2 + i\epsilon)} + \frac{C^*}{((P-k-V)^2 - m_R^2 + i\epsilon)}$$

- **Effective Lagrangian in the Medium**

$\mathcal{L}_I = -ig^*\vec{\Phi}\cdot\overline{q}\gamma^5\vec{\tau}q\,\Lambda^*$

$\star$ **J.P.B.C. de Melo, K. Tsushima, B. El-Bennich, E. Rojas and T, Frederico, PRC90 (2014) 035201**

Valence Light-front wave function in Medium (Symm. Nuclear Matter)

$$\Phi^*(k^+, \vec{k}_\perp; P^+, \vec{P}_\perp) = \frac{P^+}{m_\pi^{*2} - M_0^2} \left[\frac{N^*}{(1-x)(m_\pi^{*2} - \mathcal{M}^2(m_q^{*2}, m_R^2))} + \frac{N^*}{x(m_\pi^{*2} - \mathcal{M}^2(m_R^2, m_q^{*2}))} \right]$$

- $x = k^+/P^+$, **with** $0 \leq x \leq 1$, $\quad m_\pi^* \simeq m_\pi$
- $\mathcal{M}^2(m_a^2, m_b^2) = \frac{k_\perp^2 + m_a^2}{x} + \frac{(P-k)_\perp^2 + m_b^2}{1-x} - P_\perp^2$
- **Free Square Mass operador:** $M_0^2 = \mathcal{M}^2(m_q^{*2}, m_q^{*2})$.

- **Pion Electromagnetic Form Factor in the Breit, $q^+ = 0$:**

$$F_\pi^{*(WF)}(q^2) = \frac{1}{2\pi^3(P'^+ + P^+)} \int \frac{d^2k_\perp dk^+ \theta(k^+)\theta(P^+ - k^+)}{k^+(P^+ - k^+)(P'^+ - k^+)} \Phi^*(k^+, \vec{k}_\perp; P'^+, \tfrac{\vec{q}_\perp}{2})$$
$$\times \left(k^-_{\rm on} P^+ P'^+ - \tfrac{1}{2} \vec{k}_\perp \cdot \vec{q}_\perp (P^+ - P'^+) - \tfrac{1}{4}\, k^+ q_\perp^2 \right)$$
$$\times\ \Phi^*(k^+, \vec{k}_\perp; P^+, -\tfrac{\vec{q}_\perp}{2})$$

- $C^* \quad \Rightarrow \quad F_\pi^*(0) = 1$

- Transverse momentum probability density (π at rest)

$$f^*(k_\perp) = \frac{1}{4\pi^3 m_\pi^*} \int_0^{2\pi} d\phi \int_0^{m_\pi^*} \frac{dk^+ M_0^{*2}}{k^+(m_\pi^* - k^+)} |\Phi^*(k^+, \vec{k}_\perp; m_\pi^*, \vec{0})|^2,$$

- Integration of $f^*(k_\perp)$: Probability of the valence component in the pion

$$\eta^* = \int_0^\infty dk_\perp k_\perp f^*(k_\perp).$$

- Pion decay constant

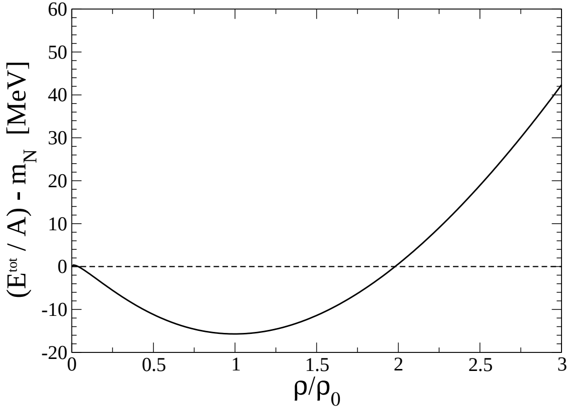
$$P_\mu \langle 0(\rho)|A_i^\mu|\pi_j^*\rangle = i m_\pi^{*2} f_\pi^* \delta_{ij} \simeq i m_\pi^2 f_\pi^* \delta_{ij}.$$

- **Pion Fock States**

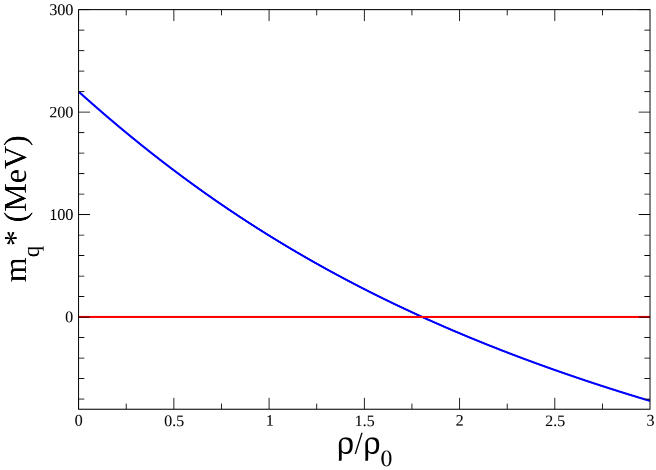
$$|\pi > \;=\; \sqrt{\eta}|q\bar{q} > +a|q\bar{q}q\bar{q} > +b|q\bar{q}g > + \cdots$$

$$|\pi >^* \;=\; \sqrt{\eta^*}|q\bar{q} >^* +c|q\bar{q}q\bar{q} >^* +d|q\bar{q}g >^* + \cdots$$

Results: Quark Meson Coupling + Light-Front



- Symmetric Nuclear Matter - Biding Energy per Nucleon
- $m_q = (5, 220)$ MeV $\rightarrow K = (279.3, 320.9)$ MeV



- **Effective mass of constituent quarks, up and down**

Pion properties in medium. η^* is the probability of the valence component in the pion. ($\rho_0 = 0.15$ fm^{-3})

ρ/ρ_0	m_q^* [MeV]	f_π^* [MeV]	$< r_\pi^{*2} >^{1/2}$ [fm]	η^*
0.00	220	93.1	0.73	0.782
0.25	179.9	80.6	0.84	0.812
0.50	143.2	68.0	1.00	0.843
0.75	109.8	55.1	1.26	0.878
1.00	79.5	40.2	1.96	0.930

- **GMOR (Gell-Mann-Oakes-Renner) Relation:**

$$\begin{aligned} m_\pi^2 f_\pi^2 &= -2m_q < \overline{q}q >, \\ m_\pi^{*2} f_\pi^{*2} &= -2m_q^* < \overline{q}q >^* \end{aligned}$$

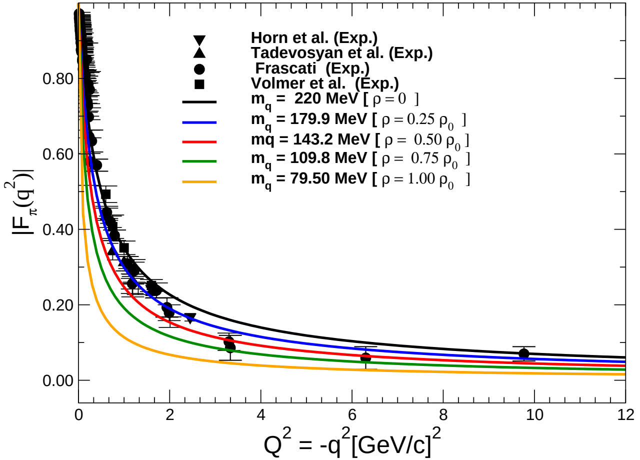
- **Vacuum quarks condensate in the medium**

$$\frac{< \overline{q}q >^*}{< \overline{q}q >} = \frac{m_q}{m_q^*}\frac{m_\pi^{*2} f_\pi^{*2}}{m_\pi^2 f_\pi^2} \simeq \frac{m_q}{m_q^*}\frac{f_\pi^{*2}}{f_\pi^2}$$

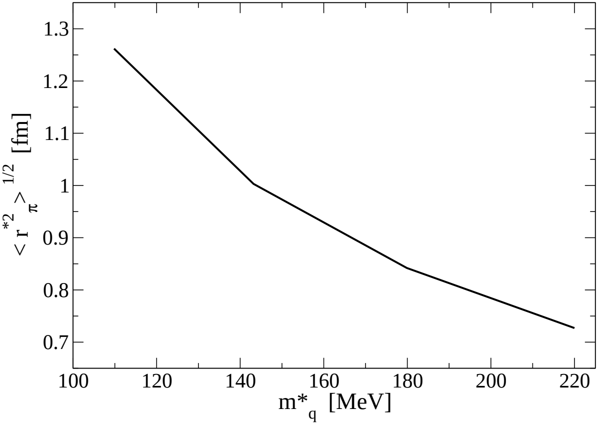
- $\rho_0 = 0.15\ fm^{-3} \Longrightarrow \approx 0.52$ (This work)
- $\rho_0 = 0.17\ fm^{-3} \Longrightarrow \approx 0.67 \pm 0.06$

Pionic Atom expt. extraction/analysis:

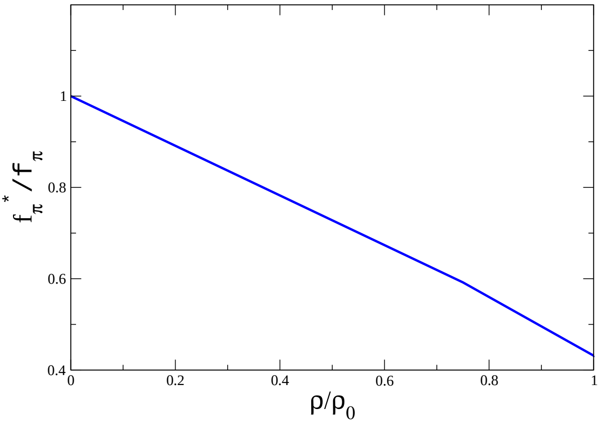
Kienle & Yamazaki, Prog. Part. Nucl. Phys. 52 (2004) 85.



- **Exp. Data (in Vacuum!!)**



- **Pion Electromagnetic Radius**



- **Pion Decay Constant**

(Valence) Wave Function, Pion // Kaon (Remind)

$$\Psi(x, k_\perp, p^+, \vec{p}_\perp) \propto \left[\frac{1}{(1-x)(m_{0^-}^2 - \mathcal{M}^2(m_q^2, m_R^2))} + \frac{1}{x(m_{0^-}^2 - \mathcal{M}^2(m_R^2, m_{\bar{q}}^2))}\right] \frac{1}{m_{0^-}^2 - \mathcal{M}^2(m_q^2, m_{\bar{q}}^2)} + [q \leftrightarrow \bar{q}]$$

here:

$$\mathcal{M}^2(m_a^2, m_b^2) = \frac{k_\perp^2 + m_a^2}{x} + \frac{(p-k)_\perp^2 + m_b^2}{(1-x)} - p_\perp^2$$

- **In the case of quarks mass** $\Rightarrow$ **Free Mass Operador**
- m_{0^-} $\Longleftrightarrow$ **Mass of the bound state**

Distribution Amplitude (normalized with f_{ps} !!!)

Def.: DAs

$$\phi_{DA}(x) = \int \frac{d^2 k_\perp}{(16\pi^3)} \Psi_{ps}(x, \vec{k}_\perp)$$

$$\int_0^1 dx \int \frac{d^2 k_\perp}{16\pi^3} \Psi_{ps}(x, \vec{k}_\perp) \ = \ \frac{f_{ps}}{2\sqrt{6}}$$

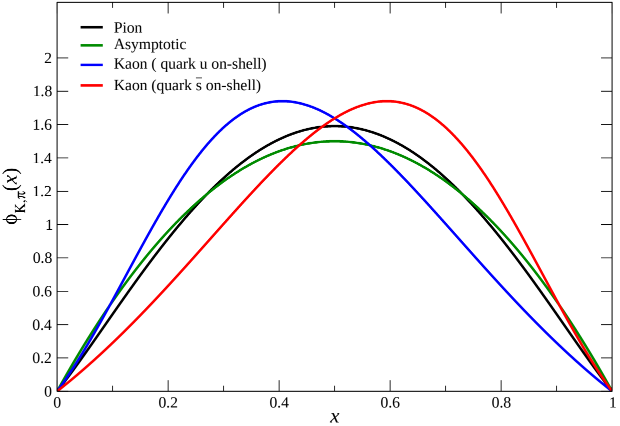
Def.: NDA (normalized to unity)

$$\phi(x) = \frac{2\sqrt{6}}{f_{ps}} \int \frac{d^2 k_\perp}{(16\pi^3)} \Psi_{ps}(x, \vec{k}_\perp) \ .$$

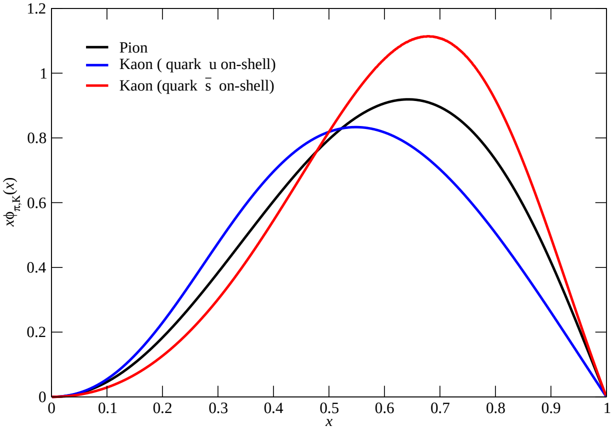
- **Pion Asymptotic**

$$\phi_\pi^{as}(x, \mu^2) \ \propto \ 6x(1-x)$$

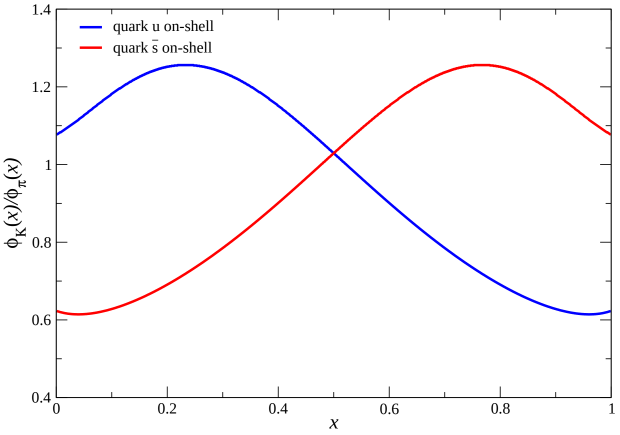
NDAs vacuum


Pion
Asymptotic
Kaon (quark u on-shell)
Kaon (quark $\bar{s}$ on-shell)
$\phi_{K,\pi}(x)$
x
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

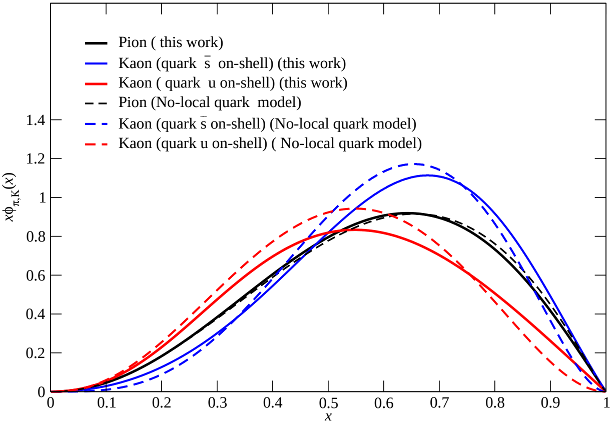
x times NDAs vacuum



Ratios: K/π NDAs vacuum



x times NDAs vacuum



- **Ref.: S. Nam, C. W. Kao, Phy. Rev. D85 (2012) 094023 ibid. Phy. Rev. D86 (2012) 074005**

(Valence) Wave Function, Pion // Kaon (Remind)

$$\Psi(x, k_\perp, p^+, \vec{p}_\perp) \propto \left[\frac{1}{(1-x)(m_{0^-}^2 - \mathcal{M}^2(m_q^2, m_R^2))} + \frac{1}{x(m_{0^-}^2 - \mathcal{M}^2(m_R^2, m_{\bar{q}}^2))}\right] \frac{1}{m_{0^-}^2 - \mathcal{M}^2(m_q^2, m_{\bar{q}}^2)} + [q \leftrightarrow \bar{q}]$$

here:

$$\mathcal{M}^2(m_a^2, m_b^2) = \frac{k_\perp^2 + m_a^2}{x} + \frac{(p-k)_\perp^2 + m_b^2}{(1-x)} - p_\perp^2$$

- **In the case of quarks mass** $\Rightarrow$ **Free Mass Operador**
- m_{0^-} $\Longleftrightarrow$ **Mass of the bound state**

Remind: Distribution Amplitude

Def.: DAs

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$$\int_0^1 dx \int \frac{d^2 k_\perp}{16\pi^3} \Psi_{ps}(x, \vec{k}_\perp) \;=\; \frac{f_{ps}}{2\sqrt{6}}$$

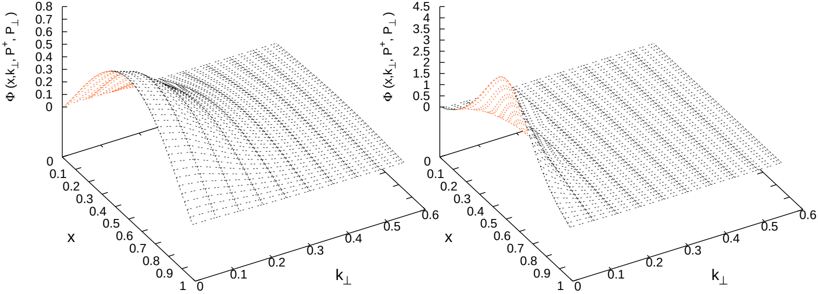
Def.: NDAs (normalized to unity)

$$\phi(x) = \frac{2\sqrt{6}}{f_{ps}} \int \frac{d^2 k_\perp}{(16\pi^3)} \Psi_{ps}(x, \vec{k}_\perp) \;.$$

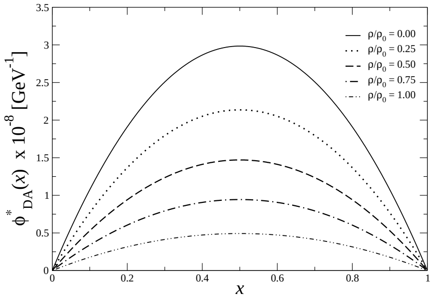
- **Pion Asymptotic**

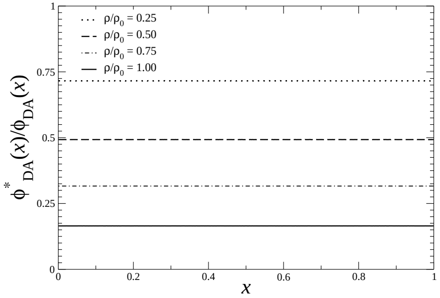
$$\phi_\pi^{as}(x, \mu^2) \;\propto\; 6x(1-x)$$

Pion (Valence) W. Func.: Vacuum (left) ρ_0 (right) $f_\pi^*/2\sqrt{6}$ normalization

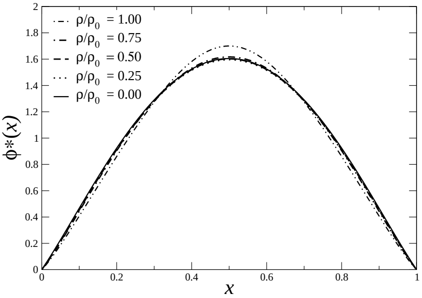


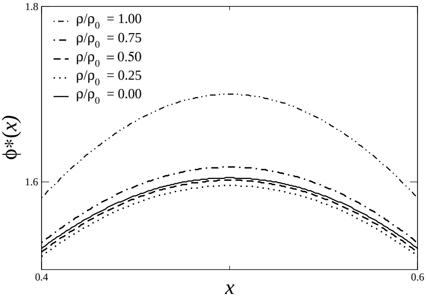
In-Medium DAs





In-Medium NDA





- Transverse momentum probability density (π at rest)

$$f^*(k_\perp) = \frac{1}{4\pi^3 m_\pi^*} \int_0^{2\pi} d\phi \int_0^{m_\pi^*} \frac{dk^+ M_0^{*2}}{k^+(m_\pi^* - k^+)} |\Phi^*(k^+, \vec{k}_\perp; m_\pi^*, \vec{0})|^2,$$

- Integration of $f^*(k_\perp)$: Probability of the valence component in the pion

$$\eta^* = \int_0^\infty dk_\perp k_\perp f^*(k_\perp).$$

- Pion decay constant

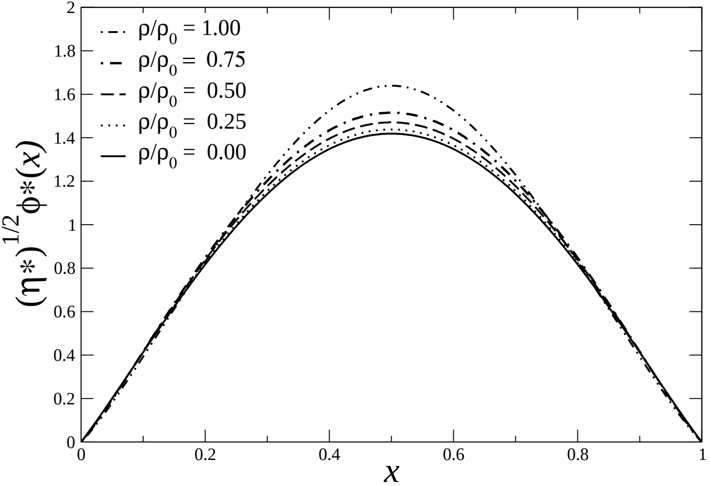
$$P_\mu \langle 0(\rho)|A_i^\mu|\pi_j^*\rangle = i m_\pi^{*2} f_\pi^* \delta_{ij} \simeq i m_\pi^2 f_\pi^* \delta_{ij}.$$

- **Pion Fock States**

$$|\pi> \;=\; \sqrt{\eta}|q\bar{q}> + a|q\bar{q}q\bar{q}> + b|q\bar{q}g> + \cdots$$

$$|\pi>^* \;=\; \sqrt{\eta^*}|q\bar{q}>^* + c|q\bar{q}q\bar{q}>^* + d|q\bar{q}g>^* + \cdots$$

In-Medium Effective NDAs



Prospects, Plans

$\Longrightarrow$ • kaon + another pseudo-scalar particles e.g., D meson

$\Longrightarrow$ • Vector particles, e.g., ρ meson [J.P.B.C. de Melo]

$\Longrightarrow$ • Nucleon, Hyperons in Light-Front

$\Longrightarrow$ • Also In Medium

Thank You Very Much !!!
LC 2016 Organizers !!!

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- **FAPESP, CNPq, CAPES**

References:

J. P. B. C de Melo, K. Tsushima, I. Ahmed, arXiv:1608.03858
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AIP Conf.Proc. 1735 (2016) 080006, arXiv:1511.09219 [hep-ph]
Phys.Rev. C90 (2014) no.3, 035201
K. Saito, K. Tsushima and A. W. Thomas,
Progress Part. Nucl. Phys. 58, 1 (2007)